University of Illinois | **First Hour Exam** | **ECE 313 Spring 2002**

1. **(10 points)** An urn contains 3 red balls and 3 black balls. A and B take turns drawing balls at random from the urn, with A drawing first. Balls that have been drawn are not replaced in the urn. What is the probability that A draws a red ball before B does?

2. **(21 points)** Events A and B are defined on a sample space Ω.

(a) **(15 points)** If P(A|A   B) = 3/5 and P(B|A   B) = 4/5,

find P(A ∩ B|A   B), P(A   B|A   B), and P(A   B|A   B).

(b) **(6 points)** C, D, and E are events of nonzero probability defined on a sample space Ω. Which of the following statements are true? You need not show any work, but to penalize guessing, 2 points will be deducted from your total score for each wrong answer.

TRUE	FALSE	
☐	☐	P(E|C D) = P(E|C) + P(E|D) – P(E|C ∩ D)
☐	☐	If P(C) = P(D), then P(C|D) = P(D|C)
☐	☐	If P(E|D) = P(D|E), then P(D) = P(E)

3. **(20 points)** Two of the five letters in a road sign that reads MIAMI have fallen down. Assume that each pair of letters is equally likely to have fallen down. A drunk randomly puts the fallen letters back into the two empty slots, possibly interchanging the positions of the letters, and possibly putting the letters back upside down. Thus, all eight possibilities corresponding to the three binary choices

{letters put back in correct position or interchanged}
{left-hand letter upside down or rightside up}
{right-hand letter upside down or rightside up}

are equally likely.

(a) **(10 points)** What is the probability that the sign still seems to read MIAMI?

(b) **(10 points)** Given that the sign still seems to read MIAMI, what is the probability that the 2 M's fell down?

4. **(30 points)** Section A of a course has 30 students of which 10 are excellent, 15 are good, and 5 are average. Section D of the course has 50 students of which 15 are excellent, 20 are good, and 15 are average.

(a) **(6 points)** One of these 80 students is picked at random and the student's classification (excellent/good/average) is determined. Let H_0 denote the hypothesis that the student is from Section A and H_1 the hypothesis that the student is from Section D. Sketch the *likelihood matrix* and indicate the maximum-likelihood decision rule by shading.

(b) **(8 points; 2 + 2 + 4)**
What is the probability that a student from Section A is assumed to be in Section D?
What is the probability that a student from Section D is assumed to be in Section A?
What is the (average) probability of making an incorrect decision?

(c) **(12 points)** Sketch the *joint probability matrix* , and show the minimum-error-probability decision rule (that is, the Bayesian or maximum *a posteriori* probability decision rule), by shading.

(d) **(4 points)** What is the probability of making an incorrect decision?
Is your answer larger, smaller or the same as the one you found in part (b)?

5. **(19 points)** Let **X** denote a geometric random variable with parameter $\frac{1}{3}$.

(a) **(10 points)** What is the average value of $(\mathbf{X}-2)^2$?

(b) **(9 points)** What is the probability that $\mathbf{X} = 2$ given that $\mathbf{X} < 4$?