1. \( P(\text{X is even}) = P(X = 2 \text{ or } 4 \text{ or } 6 \text{ or } \ldots) = q^2 + q^4 + q^6 + \ldots \) 
   \[ = \frac{q(1-q)}{1-q^2} = \frac{1}{5} \text{ which gives } q = \frac{1}{5} \text{ and } p = \frac{4}{5}. \]
   Hence, \( P(\text{X is a multiple of 3}) = P(X = 3 \text{ or } 6 \text{ or } 9 \text{ or } \ldots) = q^3 + q^6 + \ldots \) 
   \[ = \frac{q^2(1-q)}{1-q^3} = \frac{1}{3}. \]

2. Let A, B, and C respectively denote the events that the U-O, O-C, and U-C links are in working condition. The capacities are marked on the Karnaugh map below (in which each cell has probability 1/8). It is easily seen that \( Z \) takes on values 0, 56, 100, and 156 with probabilities 3/8, 1/8, 3/8, and 1/8 respectively, and hence \( E[Z] = \frac{512}{8} = 64. \)

3. The pdf is as shown on the diagram below where some lines have been added to aid in computation. Each triangle shown has area 1/8.

(a) By inspection, we see that \( P(|X| < 1/2) = 1/2. \) Similarly, \( P(X < 1/2) = 5/8 \) and \( P((X > 0) \cap (X < 1/2)) = P(0 < X < 1/2) = 1/8, \) giving \( P(X > 0 \mid X < 1/2) = 1/5. \) Politically correct anti-segregationists (i.e. those who believe in integration) who failed to sketch the pdf can proceed as follows:

\[ P(|X| < 1/2) = \int_{-1/2}^{0} u \cdot f_X(u) \cdot du + \int_{0}^{1/2} u \cdot f_X(u) \cdot du = \int_{-1/2}^{0} 1 \cdot du + \int_{0}^{1/2} u \cdot du = \frac{1}{2} + \frac{1}{2} = 1 \]

(b) \( E[X] = \int_{-\infty}^{\infty} u \cdot f_X(u) \cdot du = \int_{-1/2}^{0} u \cdot (1+u) \cdot du + \int_{0}^{1/2} u \cdot u \cdot du = \frac{u^2}{2} \bigg|^{1/2}_{-1/2} + \frac{u^3}{3} \bigg|^{1/2}_{0} = \frac{1}{6}. \)

(c) \( E[|X|] = \int_{-\infty}^{\infty} |u| \cdot f_X(u) \cdot du = \int_{-1/2}^{0} -u \cdot (1+u) \cdot du + \int_{0}^{1/2} u \cdot u \cdot du = \frac{-u^2}{2} + \frac{u^3}{3} \bigg|^{1/2}_{-1/2} + \frac{u^3}{3} \bigg|^{1/2}_{0} = \frac{1}{2}. \)

(d) \( g(u) = |u|. \) Hence, for any \( v > 0, \) we have two solutions \( u_1 = v \) and \( u_2 = -v \) to the equation \( g(u) = v. \) Since the absolute value of the derivative of \( g(u) \) is 1 at both points, we get \( f_Y(v) = f_X(v) + f_X(-v) \) for \( v > 0. \)

Obviously \( f_X(v) = f_X(-v) = 0 \) if \( |v| > 1, \) and hence we have that \( f_Y(v) = v + (1-v) = 1 \) for \( 0 < v < 1. \) In summary, \( f_Y(v) = \begin{cases} 1, & 0 < v < 1, \\ 0, & \text{otherwise}, \end{cases} \) which is readily seen to be a valid pdf.

(e) Since \( Y \) is uniformly distributed on \((0,1),\) its mean value is 1/2 (as you have written down on your sheet of notes, I hope!). Otherwise, compute \( E[Y] = \int_{0}^{1} v \cdot 1 \cdot dv = \frac{v^2}{2} \bigg|^{1}_{0} = \frac{1}{2}. \) to get the same answer as in part (c).