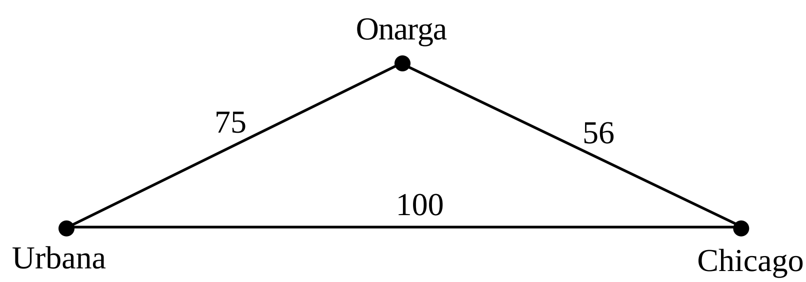
University **of Illinois** | **Second Hour Exam** | **ECE 313** **Spring 2000**

1. **(20 points)** Let **X** denote a geometric random variable with parameter p.
If P{**X** is an even number} = $\frac{1}{6}$, find the numerical value of P{**X** is a multiple of 3}.

2. **(20 points)** In the network shown below, the links are telephone trunk lines that have call-carrying capacities as marked. Each link fails with probability $\frac{1}{2}$ independently of the other links, and when it fails, its call-carrying capacity is reduced from the value marked to 0.

Let **Z** denote the call-carrying capacity of this network between Urbana and Chicago. Find the expected value of **Z**.

Remember that you are being asked about the *call-carrying* ***capacity*** of the system, that is, the **maximum** number of calls that **can** be carried, and **not** about the number of calls that **are** being carried. Also, assume that no calls originate in Onarga, and thus the entire capacity of all the trunk lines is available for carrying the Urbana-Chicago traffic.



3. **(60 points)** **X** denotes a continuous random variable with probability density function $f_X(u)$ given by

$$f_X(u) = \begin{cases} 1 + u, & -1 < u \le 0, \\ u, & 0 < u < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) **(20 points)** Find $P\{|X| < 1/2\}$ and $P\{X > 0 \mid X < \frac{1}{2}\}$.

(b) **(10 points)** Find the expected value of **X**.

(c) **(10 points)** Find the expected value of $|X|$.

(d) **(14 points)** Compute the pdf of the random variable $Y = |X|$. To obtain full credit, you must specify the value of $f_Y(v)$ for all real numbers v, $-\infty < v < \infty$.

(e) **(6 points)** Find E[**Y**] *from the pdf found in part (d)* and compare to the answer of part (c).