1.(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A|B)P(B)$.
Hence, $P(B) = [P(A \cup B) - P(A)]/[1 - P(A|B)] = 2/5$.

(b) $P(C|A \cup B) = \frac{P(C \cap (A \cup B))}{P(A \cup B)} = \frac{P((C \cap A) \cup (C \cap B))}{P(A \cup B)} = \frac{P(C \cap A) + P(C \cap B)}{P(A) + P(B)}$ since A and B are disjoint. Now, $P(C \cap A) = P(C|A)P(A)$, $P(C \cap B) = P(C|B)P(B)$, and $P(A) = 2P(B)$ giving that $P(C|A \cup B) = \frac{P(C|A)P(A) + P(C|B)P(B)}{P(A) + P(B)} = \frac{P(C|A)2P(B) + P(C|B)P(B)}{2P(B) + P(B)} = \frac{2P(C|A) + P(C|B)}{3} = \frac{8}{21}$

2. The probability that the alarm will ring is $\frac{9}{10}\times\frac{2}{3} = \frac{3}{5}$, and thus P(no ring) = $\frac{2}{5}$. It follows that

$P\{\text{wake on time}\} = P\{\text{wake}|\text{no ring}\}P\{\text{no ring}\} + P\{\text{wake}|\text{ring}\}P\{\text{ring}\} = \frac{1}{4}\times\frac{2}{5} + \frac{5}{6}\times\frac{3}{5} = \frac{3}{5}$.

3.(a) The likelihood matrix and maximum-likelihood decision rule is as shown below.

	Excellent	Good	Average
H_0: student is from Section A	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{1}{6}$
H_1: student is from Section D	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{3}{10}$

Sanity check: Each row adds up to 1, just as it should for a likelihood matrix. The desired conditional probabilities can be read off from the likelihood matrix as shaded.

P{student from Section A is assumed to be in Section D} = $\frac{1}{6}$

P{student from Section D is assumed to be in Section A} = $\frac{7}{10}$.

Since the probability that the student is in Section A is $\frac{3}{8}$, we get P{incorrect decision} = $\frac{1}{6}\times\frac{3}{8} + \frac{7}{10}\times\frac{5}{8} = \frac{1}{2}$.

More simply, 5 students from Section A and 35 students from Section D, that is, a total of 40 out of the 80 students, are mis-classified.

(c) The joint probability matrix and minimum-error-probability decision rule are as shown.

	Excellent	Good	Average
H_0: student is from Section A	$\frac{2}{6}\times\frac{3}{8} = \frac{2}{16}$	$\frac{3}{6}\times\frac{3}{8} = \frac{3}{16}$	$\frac{1}{6}\times\frac{3}{8} = \frac{1}{16}$
H_1: student is from Section D	$\frac{3}{10}\times\frac{5}{8} = \frac{3}{16}$	$\frac{4}{10}\times\frac{5}{8} = \frac{4}{16}$	$\frac{3}{10}\times\frac{5}{8} = \frac{3}{16}$

Sanity check: The sum of all the entries is 1. Note that we always decide the student is from Section D!

(d) P{incorrect decision} = P(chosen student is from Section A} = $\frac{3}{8} < \frac{1}{2}$, the answer in part (b). It better be!

(e) P{excellent student from A and good student from B} = $\frac{1}{3}\times\frac{2}{5} = \frac{2}{15}$

P{excellent student from D and good student from A} = $\frac{3}{10}\times\frac{1}{2} = \frac{3}{20} > \frac{2}{15}$. Hence, the maximum-likelihood decision is that the excellent student is from Section D and the good student from Section A, i.e. H_1.

4.(a) A binomial random variable with parameters (n,p) has mean np. Hence, $E[\mathbf{X}] = 8\times\frac{1}{2} = 4$.

(b) $\mathbf{Y}$ = 1, 2, 3 passengers are left behind according as $\mathbf{X}$ = 6, 7, or 8. Since $\mathbf{X}$ takes on values 6, 7, 8 with with probabilities $\frac{28}{256}, \frac{8}{256}, \frac{1}{256}$ respectively, we readily find that $E[\mathbf{Y}] = \frac{1\times28 + 2\times8 + 3\times1}{256} = \frac{47}{256}$.