Assigned: Wednesday, March 10, 1999
Due: Wednesday, March 24, 1999
Reading: Ross, Chapter 5
Noncredit Exercises: Ross, pp. 232-237: 5-9, 12, 13, 15-19, 21,26, 29,30-34; pp. 237-241: 2, 3, 8, 12-22, 26, 29; pp. 241-243: 1-4, 9, 10, 13-16.

Problems:

1. The random variable **X** whose CDF is given in Problem 6 of Problem Set 7 is called a *mixed* random variable since it is neither discrete nor continuous — the CDF has discrete jumps as well as regions where it increases continuously. Find the expected value of **X**.

2.(a) Ross, p. 185, #6. Do not use the hint. Instead use Lemma 2.1 on page 197 (we discussed this in class)

(b) If **X** is a geometric random variable with parameter p, what is $P\{\mathbf{X} > k\}$? Use the fact that $P\{\mathbf{X} > k\} = P\{\mathbf{X} \geq k+1\}$ together with the result of part (a) to show that $E[\mathbf{X}] = 1/p$.

3. A newsboy purchases H newspapers for c_2 cents each and sells them for c_3 cents each. He can return unsold papers to the publisher for c_1 cents each. Note that $c_1 < c_2 < c_3$. The daily demand **X** for papers is a integer-valued random variable with pmf $p_{\mathbf{X}}(u)$.

(a) What is the probability that he sells all H newsapers? Express your answer in terms of $F_{\mathbf{X}}(u)$.

(b) Let **Z** denote the daily profit (in cents) that the newsboy makes. Write an expression for **Z** in terms of **X** and H

(c) Write an expression for his average daily profit. Your answer will depend on H, so call the expression for the **average** daily profit the function g(H).

(d) The newsboy has been buying H papers for some months and making an average profit g(H) each day. One day, he decides to buy one extra paper. What is the probability that he can sell this extra paper? Show that he makes an average **additional** profit of $(c_3 - c_2) - (c_3 - c_1)F_{\mathbf{X}}(H)$ from the extra paper. Call this A(H).

(e) Show that the average **additional** profit $A(H) = (c_3 - c_2) - (c_3 - c_1)F_{\mathbf{X}}(H)$ satisfies

$$\ldots A(H-1) \geq A(H) \geq A(H+1) \ldots ,$$

that is, on average, each extra newspaper brings in smaller extra profit than the previous one. This is called the law of diminishing returns.

(f) Show that for sufficiently large H, A(H) is negative so that the newsboy loses money by buying too many extra papers.

(g) How many papers should he purchase to maximize his average profit?

4. Which of the following are valid probability density functions? Assume that the functions are zero outside the ranges specified. For those which are not valid pdfs, state at least one property of pdfs which is not satisfied. Also, state whether there exists a constant C such that Cf(u) is a valid pdf even though f(u) is not.

(a) $f(u) = |u|$ for $|u| < 1$. **(b)** $f(u) = 1 - |u|$ for $|u| < 1$.
(c) $f(u) = \ln u$ for $0 < u < 1$, **(d)** $f(u) = \ln u$ for $0 < u < 2$. Hint: ln u can be integrated by parts
(e) $f(u) = 2u$ for $0 < u < 1$. **(f)** $f(u) = (2/3)(u - 1)$ for $0 < u < 3$.
(g) $f(u) = \exp(-2u)$, $0 < u < \infty$, **(h)** $f(u) = 4\exp(-2u) - \exp(-u)$, $0 < u < \infty$.

5. The random variable **X** has probability density function

$$f_{\mathbf{X}}(u) = \begin{cases} \alpha(1 - u), & 0 < u < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find $P\{6\mathbf{X}^2 > 5\mathbf{X} - 1\}$.

(b) Find $F_{\mathbf{X}}(u)$. Be sure to specify the value of $F_{\mathbf{X}}(u)$ for all u.

6. The weekly demand (measured in thousands of gallons) for gasoline at a rural gas station is a random variable $\mathbf{X}$ with probability density function

$$f_{\mathbf{X}}(u) = \begin{cases} 5(1-u)^4, & 0 < u < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Let C (in thousands of gallons) denote the capacity of the tank (which is re-filled weekly.)

(a) If C = 0.5, (i.e., the tank holds 500 gallons) and $\mathbf{X}$ happens to have value 0.68 one particular week, (e.g. 680 people show up each wanting to purchase a gallon of gas for their snowblowers or lawnmowers), can the gas station satisfy the demand that week? That is, can the gas station supply gasoline to all those who want to buy it that week?

(b) If C = 0.5 and $\mathbf{X}$ happens to have value 0.43 some other week, can the gas station satisfy the demand during this other week? That is, can the gas station supply gasoline to all those who want to buy it that week?

(c) If C = 0.5, what is the *probability* that the weekly demand for gasoline can be satisfied? Note that if your answer is (say) 0.666…, then, in the long run, the gas station can supply the weekly demand two weeks out of three.

(d) What is the minimum value of C required to ensure that the probability that the demand exceeds the supply is no larger than 10^{-5} ?