

Assigned: Wednesday, January 27, 1999

Due: Wednesday, February 3, 1999

Reading: Ross, Chapter 2.1-2.5 and 2.7, Chapter 3

Noncredit Exercises: (Do not turn these in) Ross pp. 54-61: 3-5, 8-14, 23, 27-29, 36, 38, 39, 41, 43, 45; pp. 61-64: 4-7, 11, 18

Those with the **4th** edition should try the following problems. Ross pp. 56-57: 4-7, 10, 12; p. 59: 3, 4; pp. 60-61: 1-6, 8, 9, 12, 15-17, 23, 24, 26, 27, 29, 31, 33

Problems:

1. An experiment consists of observing the contents of an eight-bit shift register. Assume that all $2^8 = 256$ bytes are equally likely to be the contents of the shift register.
- (a) Let A denote the event that the least significant bit in the shift register is a 1. What is $P(A)$?
 - (b) Let B denote the event that the shift register contains 4 0's and 4 1's. What is $P(B)$?
 - (c) What is $P(A \cap B)$? What is $P(A \cup B)$? What is the probability that exactly one of the events A and B occurs, i.e. what is $P(A \oplus B)$?

2. Ross, Problem 6, page 62 (5th edition).

3. ["Eat your broccoli, dear; it's good for you"] Your mother has bought three servings of broccoli and two servings of cauliflower for next week. She chooses the vegetable to be served each day (Monday, Tuesday, Wednesday, Thursday, and Friday) from those that she still has on hand that day. (On Friday, she has no choice; whatever didn't get served previously is what you are going to get!)

- (a) Define an appropriate sample space and state how many outcomes are in it.
- (b) What is the probability of having broccoli on Monday?
- (c) What is the probability of having broccoli on Monday and Friday?
- (d) What is the probability of having respectively broccoli, broccoli, cauliflower, broccoli, and cauliflower on Monday, Tuesday, Wednesday, Thursday, and Friday?

Optional for zero extra credit: Write a 500-word essay to explain why you like dorm food so much.

4. The experiment consists of picking a letter at random from the word CHATTANOOGA.

- (a) Define a sample space with 11 equally likely outcomes. What is the probability that the letter picked is a vowel?
- (b) Another way of setting up a probability space is to take $\Omega = \{A, C, G, H, N, O, T\}$ with the letters having unequal probabilities. What should be the probabilities assigned to these letters?

Now consider picking **three letters** at random (choosing a subset of size 3) from the letters in the word CHATTANOOGA.

- (c) What is the probability that the letters chosen can be arranged to form one of the common words CAT, HAT, OAT, TAN and ANT? (Doesn't matter which word is formed)
- (d) Repeat part (c) assuming that **sampling with replacement** is being used
- (e) Both for sampling with replacement and for sampling without replacement, find the probability that the letters, **as they are picked**, form one of the 5 words of part (c) (doesn't matter which one) **without having to be re-arranged**.

5. Consider an experiment with a finite sample space containing n equally likely outcomes. Thus, there are 2^n different events defined on this sample space.

If you cannot solve the problem for general n , solve it (for 50% credit) for the case $n = 3$ and $\Omega = \{a, b, c\}$. If you choose to solve this special case, then, in part (d), assume that outcome b occurs.

- (a) Show that 2^{n-1} events are comprised of an odd number of outcomes while $2^n - 2^{n-1}$ events are comprised of an even number of outcomes. (Zero is an even number)

- (b) Find the “average probability” of an event by adding up the probabilities of all 2^n events and dividing the resulting sum by 2^n .
- (c) How many of the 2^n different events have probability equal to the average probability that you found in part (b)?
- (d) It was noted in class that when a trial of the experiment is performed, exactly 2^{n-1} events occur while the other 2^{n-1} events do not occur. What is the average probability of the 2^{n-1} events that do occur on a given trial of the experiment?
6. The manufacturer of a cereal tests samples from the production line to see if the samples snap, crackle, and pop as advertised. Let A, B, and C denote respectively the events that the sample **does not** snap, **does not** crackle, and **does not** pop. The manufacturer's tests show that $P(A) = 0.2$, $P(B) = P(C) = 0.3$, $P(AB \cap BC \cap AC) = 0.3$, $P(ABC) = 0.05$, $P(AB) = 0.1$, and $P(BC) = 2P(AC)$.
- (a) Sketch the sample space and indicate on it the events A, B, and C.
- (b) What is the probability that the cereal snaps, crackles, and pops ?
- (c) Cereal that fails exactly one test is sold to supermarket chains at discount prices as Soggies, Bursties, and Mushies. What is the probability of the sample failing the snap test **only**? the crackle test **only**? the pop test **only**?