University of Illinois Spring 1998

ECE 313: Problem Set #5

Assigned: February 20, 1998
Due: March 2, 1998
Reading: Ross, Chapter 3

1. Let A, B, and C denote the events that your mother serves respectively asparagus, broccoli, and cauliflower for dinner. From (bitter?) experience you know that these events are disjoint, and that $P(A) = 0.2$, $P(B) = 0.5$, $P(C) = 0.3$. Your mother makes independent decisions (without taking your opinion into account!) about the vegetable to serve each day. Over a three day period, what is the probability that:

(**a**) she serves the same vegetable on all three days?

(**b**) she serves the same vegetable exactly two days out of three?

(**c**) she serves different vegetables on the three days?

(**d**) she served broccoli on the first day, given that all the three different vegetables were served during the three days?

2. Suppose that 75 passengers hold reservations for a flight from Chicago to Champaign. The airplane has 70 seats only. Each passenger decides independently with probability 0.8 whether to show up for the flight.

(**a**) Find the probability that all passengers who show up get seats.

(**b**) Suppose that 10 passengers are arriving in Chicago on a connecting flight which is late with probability 1/4. If the connecting flight is on time, all 10 show up for the flight to Champaign; otherwise none of the 10 shows up. The remaining 65 passengers decide independently as before (and also independently of the fate of the connecting flight). What is the probability that all passengers who show up get a seat? Given that all the passengers who showed up got a seat, what is the (conditional) probability that the connecting flight was late?

3. Alice, Bob, and Carol play a game in which each player rolls a pair of fair dice in turn, with Alice going first, then Bob, and then Carol, and then Alice again, and so on. When it is Alice's turn to play, the game terminates with a win for Alice if the sum of the two dice she rolls is 8. When it is Bob's turn, the game terminates with a win for Bob if the sum of the two dice he rolls is 9. When it is Carol's turn, the game ends with a win for Carol if the sum of the dice she rolls is 10. The game continues with each player rolling the dice in turn, until one of the three players wins. Find the win probabilities for each player.

4. Suppose that n independent trials of an experiment are performed, and that event A occurs with probability $P(A) = p$ on each trial. Let P_n denote the probability that A occurs an even number of times on n trials.

(**a**) Show that $P_n = p(1 - P_{n-1}) + (1 - p)P_{n-1}$ for $n \geq 2$.

(**b**) Use the result of part (**a**) to prove that $P_n = 0.5 + 0.5(1 - 2p)^n$ for $n \geq 1$.

Notice that if A does not occur at all, then it occurs an even number of times, because zero is an even number. This problem is taken from Ross, theoretical exercise 15, on p. 120.

5. A fair coin is tossed 10 times. What is the probability that two *consecutive* heads do not occur among the 10 outcomes?

6. In successive rolls of a pair of fair dice, what is the probability of rolling "seven" twice (not necessarily on consecutive rolls), before rolling an even number six times (not necessarily on consecutive rolls)? This is taken from Ross, problem 74, on p. 115.