University of Illinois Spring 1998

ECE 313: Problem Set #4

Assigned: February 13, 1998
Due: February 23, 1998
Reading: Ross, Chapter 3

1. The probability that an archer hits the target when it is windy is 0.4, and when it is not windy the probability of a hit is 0.7. On any shot, the probability of a gust of wind is 0.3. Find the probability that

(a) the target is hit with a shot

(b) there was a gust of wind assuming the target was missed.

2. Ross, problem 18, p. 106.

3. Ross, problem 11, p. 104.

4. Ross, problem 14, p. 105.

5. **(a)** Suppose that an event E is independent of itself. Show that $P(E)P(E^c) = 0$.

(b) Events A and B have probabilities $P(A) = 0.3$ and $P(B) = 0.4$. What is $P(A \cup B)$ if A and B are independent? What is $P(A \cup B)$ if A and B are mutually exclusive?

(c) Now suppose that $P(A) = 0.6$ and $P(B) = 0.8$. In this case, could the events A and B be independent? Could they be mutually exclusive?

6. The senate of a certain country has 100 members, consisting of 33 Anarchists, 21 Bipartisans, 12 Conservatives, and 34 Democrats. Before each vote, the four groups caucus separately. Each group decides independently of the other groups whether to support a motion. All members of the group then vote in accordance with the caucus decision.

(a) Let A, B, C, D, respectively, denote the events that the four groups support a health care bill. Suppose that the probabilities of these independent events are:

$$P(A) = 0.8 \qquad P(B) = 0.5 \qquad P(C) = 0.5 \qquad P(D) = 0.2$$

What is the probability that the bill passes? Given that the bill passes, what is the conditional probability that the Anarchists voted for it? You should assume that a simple majority (51 or more votes) is required to pass a bill.

(b) The President vetoes the bill. Now let A, B, C, D, respectively, denote the events that the four groups support the motion to override the veto. Assume that these independent events have probabilities:

$$P(A) = 0.2 \qquad P(B) = 0.3 \qquad P(C) = 0.8 \qquad P(D) = 0.9$$

What is the probability that the motion to override the veto passes. You should assume that a two-thirds majority (67 or more votes) is required to override a veto.

7. A plant containing five machines M_1, M_2, M_3, M_4, M_5 can manufacture IC's in one of three possible ways:

- pass the silicon wafer through M_1, and then through either M_2 or M_3;
- pass the silicon wafer through M_4;
- pass the silicon wafer through M_5.

Assume that each of the five machines fails independently with probability p. Find

(a) the conditional probability that the plant can manufacture IC's if M_4 has failed;

(b) the unconditional probability that the plant can manufacture IC's;

(c) the conditional probability that M_4 has failed, if the plant can manufacture IC's.