University of Illinois Spring 1998

ECE 313: Problem Set #2

Assigned: January 30, 1998
Due: February 11, 1998
Reading: Ross, Chapters 1 and 2

1. An ice cream store manufactures five basic flavors (vanilla, chocolate, mint, strawberry, and fudge) of ice cream, and then creates its specialty flavors by mixing two or more basic flavors in equal proportions.

(a) How many specialty flavors can be obtained by mixing three basic flavors? How many of these contain chocolate? How many of these contain either chocolate or fudge or both? How many of these contain either chocolate or fudge but not both?

(b) How many distinct flavors (including basic and specialty flavors) of ice cream can the store offer?

Optional for zero additional credit: Identify the store!

2. Express each of the following events in terms of the events A, B, and C, and the operations of complementation, union, and intersection:

(a) at least one of the events A, B, C occurs;

(b) at most one of the events A, B, C occurs;

(c) none of the events A, B, C occurs;

(d) all three events A, B, C occur;

(e) exactly one of the events A, B, C occurs;

(f) events A and B occur, but not C;

(g) either event A occurs, or if not then B also does not occur.

In each case draw the corresponding Venn diagrams or Karnaugh maps, whichever is more convenient.

3. Suppose that A and B are two events defined over the same sample space, with probabilities $P(A) = 3/4$ and $P(B) = 3/8$.

(a) Show that $P(A \cup B) \geq 3/4$.

(b) Show that $1/8 \leq P(AB) \leq 3/8$.

(c) Give inequalities analogous to **(a)** and **(b)** for $P(A) = 2/3$ and $P(B) = 1/2$.

4. Consider an experiment with a finite sample space Ω consisting of n outcomes, all equally likely. Thus there are 2^n distinct events defined on this sample space.

(a) Show that 2^{n-1} events contain an odd number of outcomes and 2^{n-1} events contain an even number of outcomes.

(b) Find the "average probability" of an event by adding up the probabilities of all the 2^n events and dividing the result by the total number 2^n of events.

(c) How many of the 2^n events have probability exactly equal to the "average probability" that you found in part **(b)**?

5. The experiment consists of picking a student from the set of all students registered on the UIUC campus this semester. It is *not* necessary to assume that all students are equally likely to be picked, but you may make this assumption if you like.

(a) Consider the two events:

A — the student has had four years of high school science (FYS),
B — the student has had calculus in high school.

If the probability that (s)he has had at least one of FYS and calculus is 0.7, and the probability that (s)he has missed at least one of the two is 0.8, what is the probability that (s)he has had *exactly* one of the two?

(b) Let C denote the event that the student is registered in ECE313 this semester, and let events A and B and their probabilities be as in part **(a)**. If students who had had at most one of FYS and calculus did not register in EE313 this semester,

i. What is $P(A^c \cap B^c \cap C^c)$?
ii. What is the probability that the student picked is not registered in ECE313 *and* has had exactly one of FYS or calculus in high school?

(c) Using the data given in parts **(a)** and **(b)**, which of the following probabilities:

$$P(ABC),\ P(C),\quad P(A^cBC^c),\quad P(A),\quad P(A^cB^cC),\quad P(A\cup B\cup C^c)$$

can you compute? It is not necessary to actually compute each probability.

6. Let $\Omega = \{1, 2, 3, \ldots\}$ be the countably infinite sample space whose elements (outcomes) are the positive integers. For each positive integer n, define the event

$$A_n = \{\ k\ :\ k \text{ is a multiple of } n\ \}$$

(a) Find n and m such that $A_n = A_3 \cap A_4$ and $A_m = A_6 \cap A_9$.

(b) If $P(\{k\}) = \frac{1}{3}\cdot\left(\frac{2}{3}\right)^{k-1}$ find the probability of the event A_3.

Note: an exact answer of the form a/b, where a and b are integers, is required; answers of the form 0.210526... will receive no credit.