University of Illinois

Problem Set 1

ECE 313
Spring 1998

Assigned : Friday, January 23, 1998
Due : Monday, February 2, 1998
Reading : Ross, Chapter 1.1–1.4, Chapter 2.1–2.5
About the problems : These problems are based entirely on material covered in the *prerequisite* (only calculus) to this course. The problems are meant as a self-diagnosis aid; this material underlies most of the mathematical development in this course.

1. In this problem all angles are expressed in **radians**.

   (a) Use your calculator to evaluate $\sqrt{52}\cos\left(3^{-1}\arctan\left(\frac{18\sqrt{3}}{35}\right)\right)$.

   (b) In this part you have to find the limit of $\frac{1}{\sin^2 x} - \frac{1}{\tan^2 x}$ as $x \to 0$. Use your calculator to evaluate this function for *small* values of x, say $x = 0.1, 0.01, 0.001, \ldots$. Does this function seem to be approaching a limit? If so, what do you think it is? Now, use what you have learnt about limits in calculus to find $\lim_{x\to 0}\left(\frac{1}{\sin^2 x} - \frac{1}{\tan^2 x}\right)$ analytically.

   (c) Repeat the above procedure for the function $f(x) = \frac{1}{\sin^2 x} - \frac{1}{x^2}$. (Hint: the answer is not $0, 1$ or ∞.)

2. Find the maxima (plural) of $f(x) = x^4(1.001)^{-x^2}$ on the real line. What about the maxima of $f(x) = x^4(1.001)^{x^2}$?

3. Find the values of the following **definite** integrals:

   (a) $\int_{-1}^{1} |x|\, dx$ (b) $\int_{0}^{1} x(1-x^2)^{11}\, dx$.

   (c) $\int_{-\infty}^{\infty} x\exp(-x^2/2)\, dx$. (Hint: Plotting the integrand might give you some clues.)

4. Does there exist a function $f(x)$ satisfying the following two conditions : **(i)** $f(x) \geq 0$ for all real numbers in the range $a \leq x \leq b$, and **(ii)** $\int_a^b f(x)\, dx < 0$? (For example, does this hold for the integrals in part (a) of the previous questions?)

5. Let $\frac{d}{dx}f(x) = g(x)$, $-\infty < x < \infty$. Which of the following statements is true for all x? (Here, C denotes an arbitrary constant.)

   (i) $\frac{d}{dx}f(-x) = -g(-x)$ (ii) $\frac{d}{dx}f(x^2/2) = xg(x^2/2)$ (iii) $\frac{d}{dx}\exp\left(f(x^2)\right) = g(x^2)\exp\left(f(x^2)\right)$

   (iv) $\int g(-x)\, dx = f(-x) + C$ (v) $\int g(x^2/2)\, dx = \frac{f(x^2/2)}{x} + C$ (vi) $\int \frac{g(x)}{f(x)}\, dx = \ln(f(x)) + C$

6. Evaluate the following **definite** two-dimensional integrals over the specified domains of integration:

   (a) $f(x,y) = \min(x,y)$, over the region $\{(x,y) : 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$.

   (b) $f(x,y) = (x^2+y^2)^{-2}$, over the region $\{(x,y) : x^2+y^2 > 1\}$.

7. **Super-Calvinball:** Calvin and Hobbes start running towards each other from opposite ends of a 100 meter long field, each running at a speed of $1m/sec$ (yeah, we're not talking world record speeds here). As they start, Calvin throws a baseball he's holding to Hobbes, who catches it in midstride and throws it back without wasting a second. Calvin, not to be outdone, catches the ball (in midstride, and without wasting any time either!) and throws it back, ad nauseum. They both throw the ball at a speed of $2m/sec$. If they continue this process till they collide at the center of the field (where they sit dazed for a minute or two before resuming Super-Calvinball), write an expression for how many meters the ball has travelled. Do you recognize this series? What does the expression simplify to? (What does this tell you about the relationship between the length of a question and its answer?!)