University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 313
Hour Exam # 1

Wednesday, March 4, 1998
7:00 p.m. — 8:00 p.m.
Room 112 Chemistry Annex Building

Name ______________________________

University ID Number ______________________________

Signature ______________________________

Section:

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INSTRUCTIONS

This exam is closed book and closed notes, except that one $8.5'' \times 11''$ sheet of notes (both sides) is allowed. Calculators, laptop computers, tables of integrals, etc. may not be used.

The exam consists of 4 problems worth a total of 120 points. Of these, 20 points in Problem 4 are **extra-credit points**. That is, a score of only 100 points is needed to receive the full 20% credit given to the first hour exam. Scores in excess of 100 points will compensate for other Exam and Homework scores.

Write your answers in the spaces provided. Show *all* your work, except in the true/false problem. If you need extra space, please use the back of the previous page. Partial credit will be given only for substantial progress on a problem.

Grading

1.	24 points	________
2.	17 points	________
3.	32 points	________
4.	27 points	________
extra:	20 points	________
	TOTAL	________

Problem 1 (24 points)

The following are statements about events A, B, and C with nonzero probabilities $P(A)$, $P(B)$, and $P(C)$. The events are arbitrary, unless a specific condition is specified in the statement. Answer True if the statement is true for **all** such events A, B, C. Otherwise, answer False.

True	False	
☐	☐	if $A \subset B$, then $P(B\|A^c) = 0$
☐	☐	$P(A \oplus B) = P(A\|B^c)P(B^c) + P(A^c\|B)P(B)$
☐	☐	$P(AB) + P(AC) + P(BC) - P(ABC) = P(ABC^c) + P(AB^cC) + P(A^cBC)$
☐	☐	if $P(A) = P(B)$, then $P(A\|B) = P(B\|A)$
☐	☐	if $P(A\|B) > P(B\|A)$, then $P(A) > P(B)$
☐	☐	if $P(A\|B) = P(B\|A)$, then A and B are independent
☐	☐	if $P(ABC) = P(A)P(B)P(C)$, then A, B, C are independent events
☐	☐	if A, B, C are independent, then $P(A \cup B \cup C) + P(A^c)P(B^c)P(C^c) = 1$

Check the appropriate box for each of the eight statements above. **No justification is required**. However, so as not to give points for random guessing, you will receive +3 points for correct answer, 0 points for no answer, and −3 points for wrong answer.

Problem 2 (17 points)

A student is taking a multiple choice exam with 4 alternatives for each question. If the student knows the answer, she selects it (and is always right). Otherwise, she selects at random from the 4 possible answers. Suppose that the student knows the answers to 70% of the questions.

a. What is the probability that she selects the correct answer to a given question?

$$\boxed{P(\text{correct}) \ =}$$

b. If the student selects the correct answer, what is the conditional probability that she actually knows the answer?

$$\boxed{P(\text{knows}|\text{correct}) \ =}$$

Problem 3 (32 points)

Alice and Bob play the following game of dice. Each round of the game consists of Alice rolling one die and Bob rolling two dice. All the dice are fair. Here are the rules of this game:

- if Alice's number falls strictly between the two numbers rolled by Bob, then Alice wins and the game stops;
- if Alice's number is equal to one of the numbers rolled by Bob, then it is a tie and the round is repeated until there is a winner;
- in all other cases, Bob wins and the game stops.

For example: if Alice rolls (3) and Bob rolls $(1,4)$ or $(4,1)$, then Alice wins; if Alice rolls (3) and Bob rolls $(1,3)$, then there is a tie; if Alice rolls (3) while Bob rolls $(1,1)$, then Bob wins. Based on these rules, what is the probability that:

a. Alice wins on the first round, given that she rolls a (3)?

$P(A|3) =$

b. There is a tie on the first round?

Alice wins on the first round?

Bob wins on the first round?

$P(T) =$ $P(A) =$ $P(B) =$

c. Alice wins the game?

$$\boxed{P(\text{Alice wins}) \ =}$$

Problem 4 (27 points)

Alice, Bob, and Carol attend a dinner party together with $n-3$ other people. Upon entering, all the diners hand over their business cards to the doorman for drawing in a lottery. There are **two prizes** in the lottery. Thus after the dinner is over, two cards are drawn at random (one after the other, without replacement) and their bearers each win a prize.

a. What is the probability that Alice wins a prize?

$P(A) =$

Next week Alice, Bob, and Carol attend this dinner party again. There are again $n-3$ other people at this party, and exactly the same lottery is supposed to take place. However, this time around Bob and Carol find out, **just after the first card is drawn**, that the doorman forgot to put their cards in the draw-basket. After much argument, the lottery organizers agree to put **two cards each** for Bob and Carol in the draw-basket before the second card is drawn.

b. What is the probability that Bob wins a prize?

$P(\mathcal{B}) =$

c. What is the probability that Alice wins a prize?

$P(\mathcal{A}) =$

Who has better chances of winning a prize: Alice or Bob?

☐ Alice ☐ Bob ☐ same

Express your answers in terms of the number n of diners at the party, and show all your work.

Extra-credit (20 points): Just before Bob and Carol put cards in the draw-basket, an engineering student from U of I walks up and says that she knows how to fix the problem in a jiffy. By making a simple modification of the lottery rules and procedures, that only takes seconds to implement, she equalizes all the diners' probabilities of winning a prize.

What modification did she propose?

State your answer precisely, and show why it works!