University of Illinois Fall 2017

ECE 313: Hour Exam II

Wednesday, November 15, 2017
8:45 p.m. — 10:00 p.m.

Name: (in BLOCK CAPITALS) ________________________________

NetID: ______________ **Signature:** ______________________

Section:
☐ A, 9:00 a.m. ☐ B, 10:00 a.m. ☐ C, 11:00 a.m. ☐ D, 1:00 p.m. ☐ E, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5”×11” sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, cellphones, headphones, etc. are not allowed.

The exam consists of **nine** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 6 points ______________
2. 10 points ______________
3. 16 points ______________
4. 16 points ______________
5. 22 points ______________
6. 16 points ______________
7. 14 points ______________

Total (100 points) ____________

1. **[6 points]** Suppose X and Y have the joint pdf:

$$f_{X,Y}(u,v) = \begin{cases} C & u^2 + v^2 \leq 1 \\ 0 & \text{else,} \end{cases}$$

(a) Find C.

(b) Are X and Y independent? Explain why.

2. **[10 points]** Suppose that buses are scheduled to arrive at a bus stop at noon but are always X minutes late, where X is an exponential random variable with rate $\lambda = 1/5$. Suppose that you arrive at the bus stop precisely at noon. *NOTE:* All answers can be left expressed in terms of exponentials, e.g., ae^{-b}.

   (a) Compute the probability that you have to wait for more than 10 minutes for the bus to arrive.

   (b) Suppose that you have already waited for 10 minutes. Compute the probability that you have to wait an additional 2 minutes or more.

3. **[16 points]** The number of failures occurring in a particular wireless network over the time interval $[0, t)$ can be modeled as a Poisson process $\{N(t), t \geq 0\}$. On average, there is a failure every 4 days, i.e., the intensity of the process is $\lambda = 0.25/\text{day}$. *NOTE:* All answers can be left expressed in terms of sums of exponentials, e.g., $ae^{-b} + ce^{-d}$.

   (a) What is the probability of at most 1 failure in $[0, 8)$ and at least 2 failures in $[8, 16)$? The given time intervals are in days.

   (b) Let T_3 be the time of the third failure. Compute $P(T_3 > 8)$ (time unit: days).

4. **[16 points]** Suppose the output transmission power of a cellular phone is X dBm (decibel-milliwatts), where X is uniformly distributed over the interval $[20, 30]$. Then $Y = 10^{X/10}$ is the transmission power in mW (milliwatts). Find the pdf of Y.
NOTE: The answer can be left expressed in terms of logarithms like $\log_{10} e$ or $\ln 10$.

5. **[22 points]** The pdf of the Kumaraswamy distribution is

$$p_W(w) = abw^{a-1}(1-w^a)^{b-1}, \text{ where } w \in [0,1],$$

where a and b are non-negative shape parameters. We observe a signal in the presence of additive Kumaraswamy noise with $a = 2$ and $b = 2$, i.e., $Y = X + W$, where Y is the observation, X is the original signal and W is the noise. Note that W is supported on the interval $[0,1]$ and is not symmetric around $1/2$.

(a) Suppose we have two possible signals $X \in \{0, 2\}$, where $P(X = 0) = 0.3$ and $P(X = 2) = 0.7$. Design a decision rule that minimizes the probability of error.

(b) Evaluate the probability of error, which we denote by $P_e^{(b)}$.

(c) Suppose we instead have two possible signals $X \in \{0, \frac{1}{2}\}$, where $P(X = 0) = 0.5$ and $P(X = \frac{1}{2}) = 0.5$. Design a decision rule that minimizes the probability of error.

(d) Let us call the average probability of error in this case to be $P_e^{(d)}$. Is $P_e^{(d)} \geq P_e^{(b)}$, YES or NO? (Note: You do NOT need to compute $P_e^{(d)}$.)

6. **[16 points]** We have a coin with an unknown probability of showing head. We denote this unknown probability by X and we know that the pdf of X is given by

$$f_X(p) = \frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha,\beta)},$$

where $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$, and $\Gamma(n) = (n-1)!$ if n is a positive integer. We toss the coin 5 times. Let $\alpha = 2$ and $\beta = 2$. What is the probability that we observe 4 heads?

7. **[14 points]** You have two machines. Machine 1 has lifetime T_1, which is Exponential(λ_1), and Machine 2 has lifetime T_2, which is Exponential(λ_2). The lifetimes are independent random variables. Machine 1 starts at time 0 and Machine 2 starts at time T. Assume that T is deterministic. Compute the probability that Machine 1 is the first to fail.