Key discrete-type distributions

Bernoulli(p)**:** $0 \le p \le 1$

$$\text{pmf: } p(i) = \begin{cases} p & i = 1 \\ 1-p & i = 0 \end{cases}$$
$$\text{mean: } p \qquad \text{variance: } p(1-p)$$

Example: One if heads shows and zero if tails shows for the flip of a coin. The coin is called fair if $p = \frac{1}{2}$ and biased otherwise.

Binomial(n, p)**:** $n \ge 1$, $0 \le p \le 1$

$$\text{pmf: } p(i) = \binom{n}{i} p^i (1-p)^{n-i} \qquad 0 \le i \le n$$
$$\text{mean: } np \qquad \text{variance: } np(1-p)$$

Significance: Sum of n independent Bernoulli random variables with parameter p.

Poisson(λ)**:** $\lambda \ge 0$

$$\text{pmf: } p(i) = \frac{\lambda^i e^{-\lambda}}{i!} \quad i \ge 0$$
$$\text{mean: } \lambda \qquad \text{variance: } \lambda$$

Example: Number of phone calls placed during a ten second interval in a large city.

Significant property: The Poisson pmf is the limit of the binomial pmf as $n \to +\infty$ and $p \to 0$ in such a way that $np \to \lambda$.

Geometric (p) **:** $0 < p \le 1$

$$\text{pmf: } p(i) = (1-p)^{i-1} p \qquad i \ge 1$$
$$\text{mean: } \frac{1}{p} \qquad \text{variance: } \frac{1-p}{p^2}$$

Example: Number of independent tosses of a coin until heads first appears.

Significant property: If L has the geometric distribution with parameter p, $P\{L > i\} = (1-p)^i$ for integers $i \ge 1$. So L has the *memoryless property* in discrete time:

$$P\{L > i + j \mid L > i\} \quad = \quad P\{L > j\} \text{ for } i, j \ge 0.$$

Any positive integer-valued random variable with this property has the geometric distribution for some p.

Key continuous-type distributions

Gaussian or Normal(μ, σ^2) $\quad \mu \in \mathbb{R}$, $\sigma \geq 0$

$$\text{pdf : } f(u) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(u-\mu)^2}{2\sigma^2}\right) \qquad \text{mean: } \mu \qquad \text{variance: } \sigma^2$$

Notation: $Q(c) = 1 - \Phi(c) = \int_c^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$
Significant property (CLT): For independent, indentically distributed r.v.'s with mean mean μ, variance σ^2:

$$\lim_{n\to\infty} P\left\{\frac{X_1 + \cdots + X_n - n\mu}{\sqrt{n\sigma^2}} \leq c\right\} \quad = \quad \Phi(c)$$

Exponential(λ)

$$\text{pdf: } f(t) = \lambda e^{-\lambda t} \quad t \geq 0 \qquad \text{mean: } \frac{1}{\lambda} \qquad \text{variance: } \frac{1}{\lambda^2}$$

Example: Time elapsed between noon sharp and the first time a telephone call is placed after that, in a city, on a given day.

Significant property: If T has the exponential distribution with parameter λ, $P\{T \geq t\} = e^{-\lambda t}$ for $t \geq 0$. So T has the *memoryless property* in continuous time:

$$P\{T \geq s + t \mid T \geq s\} \quad = \quad P\{T \geq t\} \qquad s, t \geq 0$$

Any nonnegative random variable with the memoryless property in continuous time is exponentially distributed.

Uniform(a, b)**:** $\quad -\infty < a < b < \infty$

$$\text{pdf: } f(u) = \begin{cases} \frac{1}{b-a} & a \leq u \leq b \\ 0 & \text{else} \end{cases} \qquad \text{mean: } \frac{a+b}{2} \qquad \text{variance: } \frac{(b-a)^2}{12}$$

Erlang(r, λ)**:** $\quad r \geq 1$, $\lambda \geq 0$

$$\text{pdf: } f(t) = \frac{\lambda^r t^{r-1} e^{-\lambda t}}{(r-1)!} \qquad t \geq 0 \quad \text{mean: } \frac{r}{\lambda} \qquad \text{variance: } \frac{r}{\lambda^2}$$

Significant property: The distribution of the sum of r independent random variables, each having the exponential distribution with parameter λ. (If $r > 0$ is real valued and $(r-1)!$ is replaced by $\Gamma(r)$ the gamma distribution is obtained.)

Rayleigh(σ^2)**:** $\quad \sigma^2 > 0$

$$\text{pdf: } f(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) \qquad r > 0 \quad \text{CDF : } 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

$$\text{mean: } \sigma\sqrt{\frac{\pi}{2}} \qquad \text{variance: } \sigma^2\left(2 - \frac{\pi}{2}\right)$$

Example: Instantaneous value of the envelope of a mean zero, narrow band noise signal.

Significant property: If X and Y are independent, $N(0, \sigma^2)$ random variables, then $(X^2 + Y^2)^{\frac{1}{2}}$ has the Rayleigh(σ^2) distribution. Failure rate function is linear: $h(t) = \frac{t}{\sigma^2}$.