University of Illinois Fall 2016

ECE 313: Exam I

Wednesday, October 5, 2016
8:45 p.m. — 10:00 p.m.
A-C will go to room ECEB 1013
D-I will go to room ECEB 1015
J-Z will go to room ECEB 1002

Name: (in BLOCK CAPITALS) ____

NetID: ____

Signature: ____

Section:
☐ A, 9:00 a.m. ☐ B, 10:00 a.m. ☐ C, 11:00 a.m. ☐ D, 1:00 p.m. ☐ E, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of 6 problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 18 points ____________
2. 18 points ____________
3. 14 points ____________
4. 16 points ____________
5. 18 points ____________
6. 16 points ____________

Total (100 points) ____________

1. **[18 points]** Let X be a random variable, with pmf $p_X(k) = ck^2$, $k = 1, 2, 3$, where c is a constant.

   (a) Find c.

   (b) Calculate $P\{X = 3\}$.

   (c) Calculate $E[X]$ and $\text{Var}(X)$.

2. [**18 points**] Suppose that 103 passengers hold reservations for a 100-passenger flight. The probability that a passenger shows up at the gate is 0.95, and is independent of other passengers. Let X be the number of passengers who show up at the gate.

   (a) On average, how many passengers show up at the gate?

   (b) Find the probability that everyone who shows up gets to go. Write an expression for the solution without computing its value.

   (c) Now, let Let Y be the number of passengers who *do not* show up at the gate. Use the Poisson approximation to give an expression for $P\{Y \geq 3\}$ involving only a small number of terms.

3. [**14 points**] The two parts of this problem are unrelated.

(a) Consider tossing a fair coin 3 times. Are events $A =$ "two or more tails" and $B =$"one or two heads" independent? Justify your answer.

(b) Prove the following statement:
If A and B are independent and $B \subset A$, then either $P(B) = 0$ or $P(A) = 1$.

4. [**16 points**] Suppose players A and B play a game involving flipping a fair coin; each time the coin is flipped it shows heads with probability 1/2 or tails with probability 1/2.

   (a) Suppose player A flips the coin first, then B, then A, and so on. The winner is the first player to get a heads. Find $P\{A \text{ wins}\}$.

   (b) Consider the following variation of the original game. Suppose player A flips first, then B flips twice, then A flips twice, then B flips twice, and so on. The winner is the first player to get a heads. Find $P\{A \text{ wins}\}$.

5. **[18 points]** Consider a binary hypothesis testing problem where under H_1 the random variable X is Geometric with parameter $\frac{1}{2}$, while under H_0, it is instead uniform (its pmf is constant) on the set of positive consecutive integers $\{2, 3, 4, 5\}$.

   (a) Obtain the ML rule. To be definite, ties are broken in favor of H_1.

   (b) Under the ML rule from part (a), obtain p_{miss}.

   (c) Obtain $p_{\text{false alarm}}$ under the MAP rule if $\pi_1 = 3\pi_0$.

6. **[16 points]** Suppose you are putting together a 7 member team. You are one of the team members, but the other 6 members are to be chosen among your 6 friends from school, your 5 friends from your football team and your 3 friends from your chess club. *NOTE:* all answers to this problem can be left expressed in term of binomial coefficients.

   (a) How many distinct teams are there if the team must consist of 2 from each of these 3 groups?

   (b) Suppose now that the number of team members from each one of the three groups can be arbitrary, as long as there is at least one team member from each one of the three groups. How many distinct teams are there this time?