Name: ____

Problem 1 (25 pts) – For each of the following parts, provide a short answer in the space provided. **Show your work and a justification** for your answer to get partial credit.

Part A (12 pts):

i. Given two events A, and B how would you test if they are independent?

$P(A \cap B) = P(A).P(B)$ or $P(A|B) = P(A)$. -3

ii. How would you test if they are mutually exclusive? -3

$P(A \cup B) = P(A) + P(B)$ or $P(A \cap B) = 0$.

iii. Can an event be both mutually exclusive and independent? Explain with a coin toss example. -3

~~{T, H}~~ (-2)

$P(X_1 = T, X_2 = T) = 1/2 \cdot 1/2 = 1/4$

[possible only if $P(A) = 0$ or $P(B) = 0$] (1)

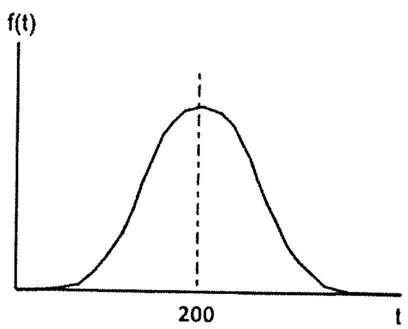
(3) There is a non-zero $P(A \cap B)$, so not possible

iv. Given three events A, B, and C, briefly but clearly state what is meant by A, B, C are pairwise independent but NOT mutually independent.

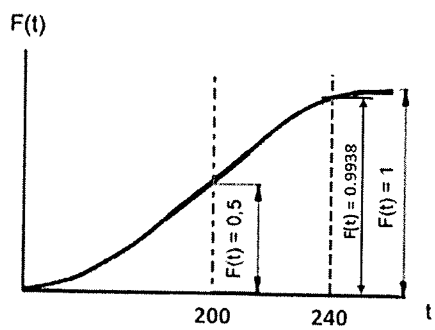
$P(A \cap B) = P(A).P(B)$, $P(A \cap C) = P(A).P(C)$, $P(B \cap C) = P(B).P(C)$

but $P(A \cap B \cap C) \neq P(A).P(B).P(C)$. -3

Part B (7 pts): Let T be a random variable representing the measurements collected from a wireless sensor. The pdf and cdf of T are plotted below, and we have: $P(X \leq 240) = 0.9938$.



pdf Distribution



Cumulative Distribution Function

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i. Is random variable T discrete or continuous?

continuous. -1

ii. Name the distribution of T:

Normal / Gaussian -1

iii. What are the parameters of the distribution of T?

Mean $\mu = 200$, $P(Z \le K) = 0.9938$ -2

$\frac{240-200}{\sigma} = 2.5$, $\sigma = 16$.

iv. Write the pmf or pdf of T =

$$P(X=x) = \frac{1}{\sqrt{2\pi \cdot 16^2}} \cdot e^{-\frac{(x-200)^2}{2\times 16^2}}$$

$$\cancel{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{16}\right)^2}}$$

-3

Part C (6 pts):

i) Suppose that the lifetime of a system is denoted by a continuous random variable X, with its corresponding pdf. Given that the system has been operational for N hours, write an expression for the probability of the system being operational for an additional time of at least 10 more hours.

$$P(X \ge N+10 \mid X \ge N) = \cancel{\frac{P(N \le X \le N+10)}{P(X \ge N)}}$$

$$\cancel{P(X \ge N+10 \mid X \ge N)}$$

-3

ii) For the same example, now you are told that X is distributed exponentially with parameter λ. Demonstrate the memoryless property, using this information.

$$\frac{P(X \ge N+10,\ X \ge N)}{P(X \ge N)} = \frac{P(X \ge N+10)}{P(X \ge N)}$$

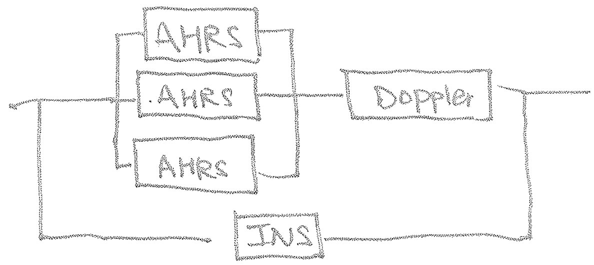
$$= \frac{e^{-\lambda(N+10)}}{e^{-\lambda N}}$$

-3

$$= e^{-10\lambda}$$

Problem 2 (20 pts) –
An aircraft can be completely navigated using an Inertial Navigation System (INS). If the INS fails, the aircraft can be navigated using the combination of the Doppler and the attitude heading and reference system (AHRS). The system contains three AHRS units, of which, **at least two** are needed.

Part A. (6 pts) Draw the reliability block diagram of the system.



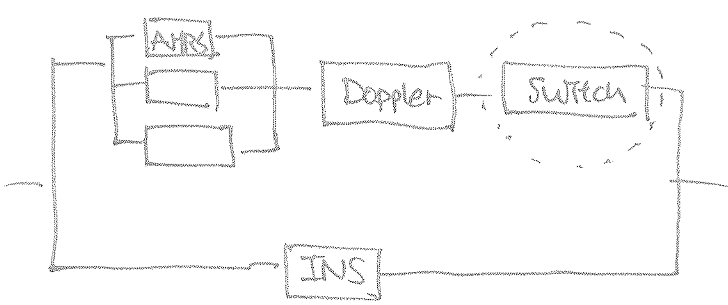
6

Part B. (7 pts) Derive an expression for the reliability of the system using the information given in the table below:

Component	Reliability
INS	(R_I)
AHRS	(R_A)
Doppler	(R_D)

$1 - (1 - R_{INS})(1 - R_D(3R_A^2 - 2R_A^3))$ -1

Part C. (7 pts) Assume that when the Inertial Navigation System (INS) fails, switching to the AHRS/Doppler system happens with a probability of p. Modify the reliability diagram in (A) and the expression for reliability in (B).



-4

$1 - (1 - R_I)(1 - R_D \cdot p \cdot (3R_A^2 - 2R_A^3)))$ -3.

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Problem 3 (15 pts) – Smart phones devices connect via a router which is connected to a Cloud system. The smart phones users periodically save their data on the cloud. The probability that any device is ready to transmit is 0.15. The router will periodically poll the phones connected to it to see if they are ready to transmit.

Let X = number of devices polled until the first ready device is located.

Part A. (6 pts) What is the probability that the first transmission does not occur in the first five polls?

$$P(X > 5) = 1 - (\underbrace{1 - (1-0.15)^5}_{P(X \leq 5)})$$

$$= (1-0.15)^5$$

$$= 0.85^5$$

6

Part B. (9 pts) What is the minimum number of devices that must be polled by the router to ensure that the probability of transmission is at least 0.75?
(*Utilize the natural logarithm table provided at the end of the exam*)

$$1 - (1-0.15)^r = 0.75$$ 4

$$0.85^r = 0.25$$

$$r \geq \frac{\log 0.25}{\log 0.85} = \frac{-1.38}{-0.16}$$

$$\geq 8.5$$ 3

minimum $r = 9$. 2

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Problem 4 (18 pts) – In mini-project 2, you implemented a detector for patient monitoring. The (not so ideal) detector you implemented can produce errors at certain instances (false alarms and miss-detections). Recall that,

a. A **false alarm** corresponds to the case where an alarm ("1") is generated based on processed input data for an interval, **given that** no patient abnormalities ("0" in the golden alarms) exist.

b. A **miss-detection** corresponds to the case where no alarm ("0") is generated based on processed input data for an interval, **given that** a patient abnormality ("1" in the golden alarms) exists. (Note that both are conditional probabilities)

The detector produces false alarms with a probability ***p*** and miss-detections with a probability ***q***. Real abnormalities in a patient ("1" in the golden alarms) occur with a probability ***r***.

Part A. (5 pts) Define the events 'There is a real abnormality' and 'The detector raises an alarm' as the events '*A*' and '*D*', respectively. Now express the probabilities *p* and *q* in terms of events '*A*', '$\bar{A}$', '*D*' and '$\bar{D}$'.

$p = P(D|\bar{A})$ – 2.5

$q = P(\bar{D}|A)$ – 2.5

Part B. (8 pts) Find the probability that the patient does not have an abnormal condition ("0" in the golden alarms), **given** the detector raised an alarm. (Note that this conditional probability is not *p*)

$P(\bar{A}|D) = \dfrac{P(\bar{A} \cap D)}{P(D)}$ – 2

$P(D) = P(A \cap D) + P(\bar{A} \cap \cancel{\bar{D}}) = p(1-r) + \cancel{qr}\ (1-q) \cdot r$ – 2

$P(\bar{A} \cap D) = \cancel{(1-q)r}\ \cancel{pr}\ p(1-r)$ – 4

Part C. (5 pts) What is the probability of error?

$P(\bar{A} \cap D) + P(A \cap \bar{D})$ – 3

$= \cancel{P(\bar{A}\ D A}$

$= P(D|\bar{A})P(\bar{A}) + P(\bar{D}|A)P(A)$

$= p(1-r) + qr$ – 2

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Problem 5 (22 pts): Recall that you implemented in Miniproject 02, a majority vote based abnormality detector using three alarm generators. In this problem, we are dealing with a slightly different situation. Suppose that there are 3 patients (A, B and C) in the ICU and each patient is attached to a monitor. For simplicity, we assume patient monitors generate alarms arbitrarily with probabilities $P_A = 0.002$, $P_B = 0.003$ and $P_C = 0.001$ during a given unit time of 1-minute. Assume that the three monitors generate alarms independent of each other.
A central unit monitoring the ICU produces a binary output every minute depending on whether or not there is an alarm (i.e., "1" if at least one alarm from A or B or C, "0" otherwise).

Part A. (6 pts)

i. What random variable describes the central monitoring unit's output?

Bernoulli

-3

ii. What is the probability that central monitoring unit's output is 1 in any given minute?

$1 - p(\bar{A} \cap \bar{B} \cap \bar{C}) = 1 - 0.998 . 0.997 . 0.999$

call this p.

-3

Part B. (16 pts) The seriousness of the patients in the ICS is determined by the number of alarms generated. As such, the physician is interested in the probability (P(X>20)) of having greater than 20 alarms in one hour (60 mins). Initially, the physician derives the probability using a binomial distribution.

i. Write the expression for P(X>20)

$$\sum_{i=21}^{60} \binom{60}{i} p^i (1-p)^{60-i}$$

-4

ii. A ECE313 student advises the physician to approximate the distribution for X as another discrete distribution. Name the distribution and write the pmf?

Poisson

-4

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iii. Write the equation for P(X>20) using one of the two distributions of your choice.

$\lambda = 60p$

$$\sum_{i=21}^{60} \frac{(60p)^i e^{-60p}}{i!}$$

4

iv. Considering the 60-minute interval, assume that there were 20 alarms in the first 45 minutes, what is the probability that the patients in the ICU will not be considered as serious.

P(0 alarms in 15 minutes)

$= \cancel{3\lambda e^{-\lambda t}}$

$\cancel{2 \cdot 15p\, e^{-15p \cdot 0}}$

$= 15p$

$$\frac{(15p)^0 e^{-15p}}{0!}$$

$= e^{-15p}$

4