University of Illinois Fall 2015

ECE 313: Hour Exam II

Wednesday, November 11, 2015
7:00 p.m. — 8:15 p.m.

Name: (in BLOCK CAPITALS) ______________________________

NetID: ______________________________

Signature: ______________________________

Section:
☐ A, 9:00 a.m. ☐ B, 10:00 a.m. ☐ C, 11:00 a.m. ☐ D, 1:00 p.m. ☐ E, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5”×11” sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of 5 problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

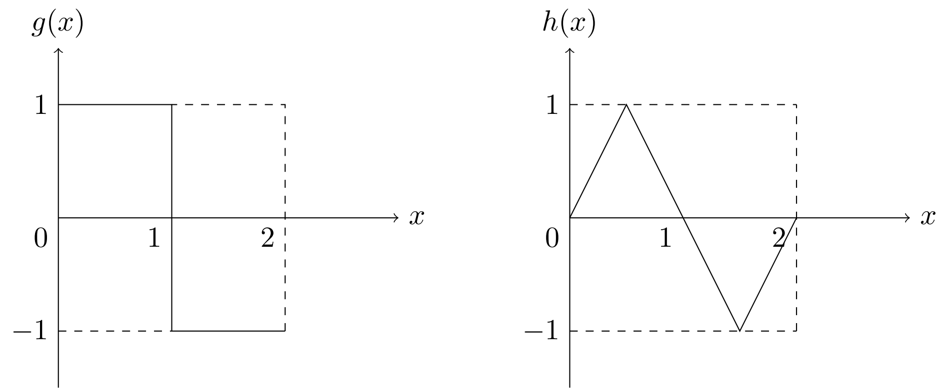
SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 20 points ____________
2. 22 points ____________
3. 20 points ____________
4. 20 points ____________
5. 18 points ____________

Total (100 points) ____________

1. **[20 points]** Let X be uniformly distributed over the interval $[0, 2]$.



(a) (5 points) Let $Y = g(X)$, where $g(x)$ is the square wave plotted above. Is the distribution of Y of discrete-type or continuous-type? If discrete, find its probability mass function (pmf); if continuous, find its probability density function (pdf).

(b) (7 points) Let $Z = h(X)$, where $h(x)$ is the triangular wave plotted above. Is the distribution of Z of discrete-type or continuous-type? If discrete, find its pmf; if continuous, find its pdf.

(c) (4 points) Find the conditional probability $P(Y > 0 \mid Z > 0)$.

(d) (4 points) Find a function r so that $r(X)$ is uniformly distributed over $[1, 5]$.

2. **[22 points]** Let N_t be a Poisson process with rate $\lambda > 0$.

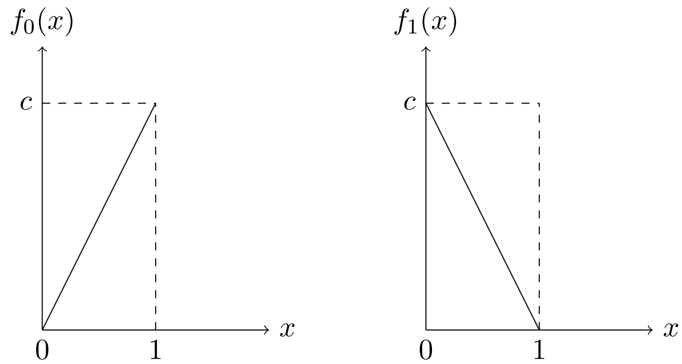
(a) (4 points) Obtain $P\{N_3 = 5\}$.

(b) (6 points) Obtain $P\{N_7 - N_4 = 5\}$ and $E[N_7 - N_4]$.

(c) (6 points) Obtain $P\{N_7 - N_4 = 5 | N_6 - N_4 = 2\}$.

(d) (6 points) Obtain $P\{N_6 - N_4 = 2 | N_7 - N_4 = 5\}$.

3. **[20 points]** Let X be a continuous-type random variable taking values in $[0, 1]$. Under hypothesis H_0, the pdf of X is f_0; under hypothesis H_1, the pdf of X is f_1. Both pdfs are plotted below. The priors are known to be $\pi_0 = 0.6$ and $\pi_1 = 0.4$.



(a) (4 points) Find the value of c.

(b) (8 points) Specify the maximum a posteriori (MAP) decision rule for testing H_0 vs. H_1.

(c) (8 points) Find the error probabilities $p_{\text{false alarm}}$, $p_{\text{miss detection}}$ and the average probability of error p_e for the MAP rule.

4. **[20 points]** Let the joint pdf for the pair (X, Y) be

$$f_{X,Y}(x,y) = \begin{cases} cxy, & 0 \le x \le 1, 0 \le y \le 1, x+y \le 1 \\ 0, & \text{otherwise,} \end{cases}$$

for some constant c.

(a) (5 points) Compute the marginal $f_X(x)$. You can leave it in terms of c.

(b) (6 points) Obtain the value of the constant c for $f_{X,Y}$ to be a valid joint pdf.

(c) (5 points) Obtain $P\left\{X + Y < \frac{1}{2}\right\}$.

(d) (4 points) Are X and Y independent? Explain why or why not.

5. **[18 points]** Suppose we are testing if a transistor in a circuit is working properly or not. The voltage we observe at the output is a normal random variable X. If the transistor is working, X is distributed according to $N(1, 1)$. If the transistor is not working, then X is distributed according to $N(-1, 1)$. There is an 50% chance that the transistor is working. You can express the answers for this problem in terms of Φ and Q.

   (a) (6 points) Find $P\{X \leq 1|\text{transistor is working}\}$.

   (b) (6 points) Find $P\{X \geq 1\}$.

   (c) (6 points) Obtain the unconditional pdf of X, $f_X(u)$ for all u.