

ECE 313: Problem Set 10
Jointly distributed random variables

Due:	Wednesday, November 20 at 6 p.m.
Reading:	<i>ECE 313 Notes</i> Sections 4.5 - 4.7
Reminder:	Exam II is on Monday, November 18, 7:00 p.m. – 8:20 p.m.
	<p>Location: Sections D (meets 11 am) and E (meets 1 pm) 100 MSEB Sections X (meets 9am) and C (meets 10am) 151 EL</p> <p>One two-sided 8.5"×11" sheet of notes allowed, with font size no smaller than 10 pt or equivalent handwriting. Bring your student ID. The exam will cover the reading assignments, lectures, and problems associated with problem sets 1-10, with emphasis on problem sets 7-10.</p>

1. **[Sums of random variables]**

Two random variables X and Y have the following joint pdf:

$$f_{X,Y}(u, v) = \begin{cases} 2 \exp(-(u + v)) & 0 < u < v < \infty \\ 0 & \text{else.} \end{cases}$$

A new random variable Z is defined as: $Z=X+Y$. Find the CDF and pdf of Z .

2. **[Joint densities-1]**

X and Y are two random variables with the following joint pdf:

$$f_{X,Y}(u, v) = \begin{cases} A(1 - |u - v|) & 0 < u < 1, 0 < v < 1 \\ 0 & \text{else} \end{cases}$$

- Find A .
- Find marginal pdfs for X and Y .
- Find $P\{X > Y\}$.
- Find $P\{X + Y < 1 | X > 1/2\}$.

3. **[Joint densities-2]**

Consider two components and three types of shocks. A type 1 shock causes component 1 to fail, a type 2 shock causes component 2 to fail, and a type 3 shock causes both components 1 and 2 to fail. The time until shocks 1, 2 and 3 occur are independent exponential random variables with rates λ_1 , λ_2 , and λ_3 , respectively. Let X_i denote the time at which component i fails, for $i = 1, 2$.

- Find $P\{X_1 > s, X_2 > t\}$.
- Find the CDF of X_1 , $F_{X_1}(s)$.

4. **[Functions of two random variables]**

Let X and Y be two independent random variables, each uniformly distributed on the interval $(0,1)$. Let $Z = \frac{1}{X+Y}$ and find the pdf $f_Z(a)$.