**University of Illinois** **Fall 2012**

# ECE 313: Problem Set 8
## Linear Scaling, Gaussian Distribution, ML Parameter Estimation

**Due:** Wednesday October 24 at 4 p.m.
**Reading:** *313 Course Notes* Sections 3.6–3.7

1. **[Random Number Generator]**
Assume that you have a random number generator that generates uniformly distributed random variables over the interval $[-3, 5]$. How would you obtain random variables that are uniformly distributed over [0,1] from the random variables that are uniform over $[-3, 5]$?

2. **[IQ]**
The intelligence quotient (IQ) of a randomly selected individual is often supposed to follow a Gaussian distribution with mean 100 and standard deviation 15.

   (a) Find the probability that an individual has an IQ above 140.

   (b) Find the probability that an individual has an IQ between 120 and 130.

   (c) Find a value $u$ such that 99% of the population has IQ at least $u$.

3. **[Random Fluctuations in a Power Source]**
A power source gives an output voltage of 12 volts. Because of random fluctuations, the true voltage at any given time is $V = 12 + 2X$, where $X \sim \mathcal{N}(0, 0.1)$. The voltage is measured once an hour (assuming independence), and if it is outside the interval [11, 13] the power source needs to be adjusted. What is the probability that no adjustment is needed during a 24-hour period?

4. **[Approximations to a Binomial Distribution]**
A communication receiver recovers a block of $n = 10^5$ bits. It is known that each bit in the block can be in error with probability $10^{-4}$, independently of whether other bits are in error.

   (a) Write down an exact expression for the probability of observing $k = 15$ errors in the block. A numerical value isn't required to be calculated.

   (b) Determine an approximate value of $P\{X = 15\}$ via the Gaussian approximation with continuity correction.

   (c) Solve part (b) using the Poisson approximation of a binomial distribution.

5. **[Gaussian Noise]**
Suppose a wireless communication system consists of a transmitter and a receiver. If the transmitter emits a radio signal with transmission amplitude $\nu$, this signal is attenuated by the air and is received by the receiver together with noise, denoted by $N$. Let $Y = \alpha\nu + N$ be the received signal amplitude, where $\alpha = 0.01$ is the attenuation factor and $N$ is a Gaussian random variable with mean $\mu = 0$ and variance $\sigma^2 = 4$.

(a) Find the pdf $f_Y(u)$ of the received signal amplitude $Y$ for $\nu = 10$.

(b) (Only for this part) Suppose that $\sigma$ is unknown, and $\nu$ and $\alpha$ are known to be 10 and 0.01, respectively. If the received signal amplitude $Y$ is 0.03, find the ML estimator $\hat{\sigma}_{ML}$ of $\sigma$.

(c) Suppose the receiver output is zero if $Y < 0$ and one if $Y \geq 0$. An error is said to occur if the receiver output is not equal to the transmitted bit. Find the probability of error, given the transmitted bit is one.