University of Illinois **Fall 2012**

ECE 313: Problem Set 1
Axioms of probability and calculating the sizes of sets

Due: Wednesday, September 5 at 4 p.m.
Reading: *ECE 313 Course Notes,* Chapter 1

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Wednesdays, and is due by 4:00 p.m. on the following Wednesday. You must drop your homework into the ECE 313 lockbox (#20) in the Everitt Laboratory basement (east side) by this deadline. Late homeworks will not be accepted without prior permission from the instructor. All assignments must be submitted stapled if they consist of more than one sheet with the following heading in block capital letters with at least 12pt size font or equivalent neat handwriting, on the top right corner of the first page:
NAME AS IT APPEARS ON COMPASS
NETID
SECTION
PROBLEM SET #
The section should be one of X,C,D,F, and the problem set number an integer. Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. **[Moderating a discussion board]**
A moderator in an online discussion board tags comments based on two criteria: whether the post was made anonymously or not, and whether the comment is relevant or irrelevant. Consider the experiment of tagging of a comment.

   (a) Define a sample space Ω describing the possible outcomes of this experiment. Explain how the elements of your set correspond to outcomes of the experiment.

   (b) What is the cardinality of Ω?

   (c) Let A be the event that the comment is relevant. Specify the outcomes in A.

   (d) Let B be the event that the comment was not made anonymously. Specify the outcomes in B.

   (e) Specify the outcomes in $A^c \cup B$.

   (f) Assuming all outcomes are equally likely, find $P(A^c \cup B)$.

2. **[A Karnaugh puzzle]**
Suppose A, B, and C are events such that: $P(A) = P(B) = P(C) = 0.5$, $P(AB) = 0.3$, $P(B \cup C^c) = 0.55$, $P(A \cup C^c) = 0.7$, and $P(ABC^c) = 2P(A^cBC^c)$. Sketch a three event Karnaugh map showing the probabilities of each one of the map sections. Show your work.

3. **[Setting up teams]**
   Suppose there are fifteen basketball players at a party, three of which are point-guards. The players are to be divided into three teams (T_1, T_2, and T_3) of five players, with all divisions being equally likely. Suppose that the labels of teams matter, so, for example, swapping all the players in one team for all the players in another team would be considered to give a different outcome.

   (a) Define a sample space Ω for this experiment.

   (b) Determine $|\Omega|$, the cardinality of Ω.

   (c) Let A be the event in which each one of the three teams has a point-guard. Find the cardinality of A.

   (d) Find $P(A)$.

   (e) Suppose now that the teams have no labels, so outcomes (T_1, T_2, T_3) and (T_2, T_1, T_3) would be indistinguishable for fixed T_1, T_2, T_3. Find $P(A)$.

4. **[Playing with cards]**
   Suppose five cards are drawn from a standard 52 card deck of playing cards, as described in Example 1.4.3, with all possibilities being equally likely.

   (a) $STRAIGHT$ is the event that the five drawn cards have consecutive values (notice that the ace can be used as lower than 2 and also as higher than K). For all of you poker players, a straight flush is to also be considered a straight. Find $P(STRAIGHT)$.

   (b) Now, suppose that a wild card is added to the deck (the wild card is allowed to represent other existing cards). Find $P(STRAIGHT)$.

5. **[Working with probability axioms]**
   Consider two events A and B.

   (a) Show that the probability that exactly one of the events A or B occurs equals $P(A) + P(B) - 2P(AB)$.

   (b) Show that $P(AB^c) = P(A) - P(AB)$.

   (c) Show that $P(A^cB^c) = 1 - P(A) - P(B) + P(AB)$.