University of Illinois Fall 2012

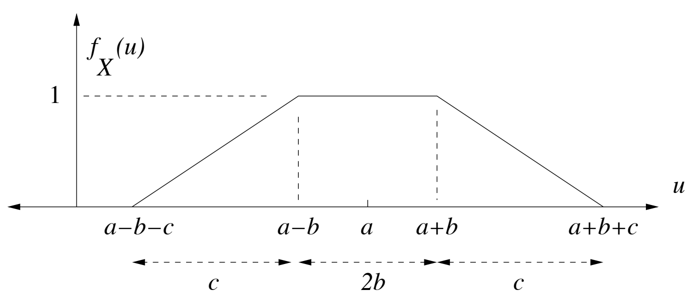
ECE 313: Hour Exam II

Monday November 12, 2012
7:00 p.m. — 8:15 p.m.
124 Burill Hall (Section X) & 100 Noyes Lab (Sections C,D,E)

1. **[14 points]** Suppose tweets from @313 follow a Poisson process with rate 1/7 per day.
   (a) (7 points) What is the probability there is exactly one tweet every week for the next four weeks?
   (b) (7 points) If there have been no tweets during two weeks of waiting, what is the mean amount of additional time until five tweets have been sent?

2. **[16 points]** Suppose a fair coin is flipped 100 times, and A is the event:
   $A = \{\ |(\text{number of heads}) - (\text{number of tails})| \geq 10\ \}$.
   (a) (6 points) Let S denote the number of heads. Express A in terms of S. Specifically, identify which values of S make A true.
   (b) (10 points) Using the Gaussian approximation with the continuity correction, express the approximate value of $P(A)$ in terms of the Q function.

3. **[14 points]** Let X be a continuous-type random variable with the pdf shown, where a, b, and c are (strictly) positive constants.



   Suppose it is known $\mu_X = 2$, and the CDF of X satisfies $F_X(u) = \frac{5}{6}(u - (a - b - c))^2$ in the interval $[a - b - c, a - b]$. Find the values of a, b, and c. Show/explain your reasoning.

4. **[16 points]** The number of months a file server is operational before it crashes is a random variable T with failure rate function given by: $h(t) = \begin{cases} 1 & \text{if } 0 < t < 1 \\ 2 & \text{if } t \geq 1 \end{cases}$
   (a) (8 points) Find the CDF of T
   (b) (8 points) Is the distribution of T memoryless? If you answer *yes*, provide a proof. If you answer *no*, provide an example showing T is not memoryless.

5. **[16 points]** Consider the hypothesis testing problem in which the pdf's of the observation X under hypotheses H_0 and H_1 are given, respectively, by:

$$f_0(u) = \begin{cases} \frac{1}{2} & \text{if } -1 \leq u \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_1(u) = \begin{cases} |u| & \text{if } -1 \leq u \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Assume the priors on the hypotheses satisfy $\pi_1 = 2\pi_0$.

(a) (8 points) Find the MAP rule.

(b) (8 points) Find $p_{\text{false alarm}}$ for the MAP rule.

6. **[24 points]** Suppose random variables X and Y have the joint probability density function (pdf): $f_{XY}(u, v) = \begin{cases} \frac{3}{2}, & u > 0,\ u^2 < v < 1 \\ 0, & \text{elsewhere} \end{cases}$

(a) (4 points) Are X and Y independent? Explain your answer.

(b) (6 points) Determine the marginal pdf of X, $f_X(u)$.

(c) (3 points) For what values of u is the conditional pdf of Y given $X = u$, $f_{Y|X}(v|u)$, well defined?

(d) (4 points) Determine $f_{Y|X}(v|u)$ for the values of u for which it is well defined. Be sure to indicate where its value is zero.

(e) (7 points) Determine $P\{Y > X\}$.