

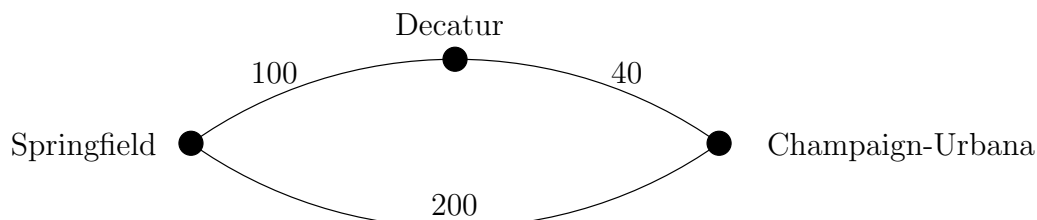
## ECE 313: Midterm Exam I

Monday October 10, 2011

6:40 p.m. — 8:00 p.m.

1320 DCL (10 and 11 am sections) and 151 Everitt (noon and 1pm sections)

1. **[20 points]** Suppose you have three fair dice.
  - (a) **[10 points]** If you roll only two of the dice simultaneously, what is the probability that the two dice show different numbers and that neither die shows 2?
  - (b) **[10 points]** If you roll the three dice simultaneously, what is the probability that all three dice show different numbers?
2. **[20 points]** Sara passes by a candy bowl filled with two types of treats: M&M bars and boxes of raisins. There are three times as many M&M bars as boxes of raisins. Sara picks up a treat at random from the bowl. She will consume the treat she picked up with probability  $\frac{1}{5}$  if the treat were an M&M bar. On the other hand, she will consume the treat she picked up with the higher probability of  $\frac{2}{5}$  if the treat were a raisin box.
  - (a) **[10 points]** What is the probability that Sara consumes the treat she picked up?
  - (b) **[10 points]** Given that Sara consumed the treat, what is the probability that the treat she picked up is a box of raisins?
3. **[20 points]** A fair die is thrown until the number 6 appears. Let  $N$  be the number of throws (including the last throw, when the number 6 appears).
  - (a) **[10 points]** Find the numerical value of the expected value,  $E[N]$ , of  $N$ .
  - (b) **[10 points]** Given that  $N \geq 3$ , what is the expected value of  $N$ ?
4. **[25 points]** A fair die is thrown until the number 6 appears twice (not necessarily in succession). Let  $M$  be the number of throws (*without* including the last throw, when the number 6 appears for the second time).
  - (a) **[10 points]** Find the probability  $M = 3$ .
  - (b) **[15 points]** Find the expected value of  $\frac{1}{M}$ .
5. **[15 points]** Messages can be transmitted between Champaign-Urbana and Springfield either directly or via Decatur on communication links whose capacities are indicated on the diagram shown below. Assume that the three links fail independently with equal probability  $\frac{1}{2}$ . Let  $\mathbb{X}$  denote the communication capacity between Champaign-Urbana and Springfield.



- (a) **[5 points]** What values can  $\mathbb{X}$  take on?
- (b) **[10 points]** Find the probability of *outage* for communication between Springfield and Champaign-Urbana. This is the probability that Springfield is unable to communicate with Champaign-Urbana via any link.