

**ECE 313: Problem Set 11**  
 Joint pdfs and functions of two random variables

<b>Due:</b>	Wednesday, November 10, at 4 p.m.
<b>Reading:</b>	<i>ECE 313 Notes</i> Sections 4.5&4.6
<b>Reminder:</b>	Hour Exam II will be held Monday November 15, 7:00 p.m. – 8:00 p.m. Section C (10 am section) Room 124 Burrill Hall, all other sections: Room 100 Noyes Laboratory One two-sided 8.5" × 11" sheet of notes allowed, with font size no smaller than 10 pt or equivalent handwriting. Bring a picture ID. The exam will cover the reading assignments, lectures, and problems associated with problem sets 1-11, with an emphasis on problem sets 7-11.

1. **[Joint densities]**

X and Y are two random variables with the following joint pdf:

$$f_{X,Y}(u,v) = \begin{cases} A(1 - |u - v|) & 0 < u < 1, 0 < v < 1 \\ 0 & \text{else} \end{cases}$$

- (a) Find A.
- (b) Find marginal pdfs for X and Y.
- (c) Find  $P\{X > Y\}$ .
- (d) Find  $P(X + Y < 1 | X > 1/2)$ .

2. **[Functions of random variables]**

Two random variables X and Y have the following joint pdf:

$$f_{X,Y}(u,v) = \begin{cases} 2 \exp(-(u+v)) & 0 < u < v < \infty \\ 0 & \text{else.} \end{cases}$$

A new random variable Z is defined as:  $Z=X+Y$ . Find CDF and pdf of Z.

3. **[Joint Densities and functions of two random variables]**

Let X and Y have joint pdf

$$f_{X,Y}(u,v) = \begin{cases} A(1 - \sqrt{u^2 + v^2}) & u^2 + v^2 < 1 \\ 0 & \text{else.} \end{cases}$$

Hint: Use of polar coordinates is useful for all parts of this problem.

- (a) Find the value of A.
- (b) Let  $Z = X^2 + Y^2$ . Find the pdf of the random variable Z.
- (c) Find  $E[Z^5]$  using LOTUS for joint pdfs:  $E[g(X, Y)] = \int \int_{\mathbb{R}^2} g(u, v) f_{X,Y}(u, v) dudv$ .

4. **[A functions of two random variables]**

Two resistors are connected in series to a one volt voltage source. Suppose that the resistance values  $R_1$  and  $R_2$  (measured in ohms) are independent random variables, each uniformly distributed on the interval (0,1). Find the pdf  $f_I(a)$  of the current I (measured in amperes) in the circuit.