

## ECE 313: Problem Set 10

## Joint distributions and independence of random variables

**Due:** Wednesday, November 3, at 4 p.m.

**Reading:** ECE 313 Notes Sections 4.1-4.4

## 1. [A joint pmf]

The joint pmf  $p_{X,Y}(u, v)$  of  $X$  and  $Y$  is shown in the table below.

	u=0	u=1	u=2	u=3
v=4	0	0.1	0.1	0.2
v=5	0.2	0	0	0
v=6	0	0.2	0.1	0.1

- Find the marginal pmfs  $p_X(u)$  and  $p_Y(v)$ .
- Let  $Z = X + Y$ . Find  $p_Z$ , the pmf of  $Z$ .
- Are  $X$  and  $Y$  independent random variables? Justify your answer.
- Find  $p_{Y|X}(v|3)$  for all  $v$  and find  $E[Y|X = 3]$ .

## 2. [A joint distribution]

Suppose  $X$  and  $Y$  are jointly continuous with joint pdf

$$f_{X,Y}(u, v) = \begin{cases} ve^{-(1+u)v} & u \geq 0, v \geq 0 \\ 0 & \text{else.} \end{cases}$$

- Find the marginal pdfs,  $f_X$  and  $f_Y$ . Note that these functions are defined on the entire real line. (Hint: To find the marginal of  $X$  it might help to review the gamma density.)
- Find the conditional pdfs,  $f_{Y|X}$  and  $f_{X|Y}$ . Be sure to indicate where these functions are well defined, and where they are zero, as well as giving the nonzero values.
- Find the joint CDF,  $F_{X,Y}(u_o, v_o)$ .
- Are  $X$  and  $Y$  independent? Justify your answer.

## 3. [Working with the joint pdf of two independent variables]

Suppose  $X$  and  $Y$  are independent random variables such that  $X$  is uniformly distributed over the interval  $[0, 1]$  and  $Y$  is exponentially distributed with parameter  $\lambda > 0$ .

- What is the joint pdf,  $f_{X,Y}$ ?
- Express  $P\{Y \geq X\}$  in terms of  $\lambda$ . Also, identify the limits of your answer as  $\lambda \rightarrow 0$  or  $\lambda \rightarrow +\infty$ .
- Express  $P\{Xe^Y \geq 1\}$  in terms of  $\lambda$ . Also, identify the limits of your answer as  $\lambda \rightarrow 0$  or  $\lambda \rightarrow +\infty$ .

## 4. [Recognizing independence]

Decide whether  $X$  and  $Y$  are independent for each of the following three joint pdfs. If they are independent, identify the marginal pdfs  $f_X$  and  $f_Y$ . If they are not, give a reason why.

- $f_{X,Y}(u, v) = \begin{cases} \frac{4}{\pi} e^{-(u^2+v^2)} & u, v \geq 0 \\ 0 & \text{else.} \end{cases}$
- $f_{X,Y}(u, v) = \begin{cases} -\frac{\ln(u)v^2}{21} & 0 \leq u \leq 1, 1 \leq v \leq 4 \\ 0 & \text{else.} \end{cases}$
- $f_{X,Y}(u, v) = \begin{cases} \frac{96u^2v^2}{\pi} & u^2 + v^2 \leq 1 \\ 0 & \text{else.} \end{cases}$