University of Illinois **Fall 2010**

ECE 313: Problem Set 7
Continuous-type random variables, uniform and exponential distributions, and Poisson processes

Due:	Wednesday, October 13, at 4 p.m.
Reading:	*ECE 313 Notes* Sections 3.2-3.5
Reminder:	Hour Exam I on Monday October 11, 7:00 p.m. – 8:00 p.m. Sections C&D Room 141 Wohlers Hall, Sections X&E Room 114, David Kinley Hall One two-sided 8.5"×11" sheet of notes allowed, with font size no smaller than 10 pt or equivalent handwriting. Bring a picture ID. The exam will cover the reading assignments, lectures, and problems associated with problem sets 1-6. The TAs will lead optional review sessions during the regular lecture times on Friday, October 8.

1. **[Continuous-type random variables I]**
   Consider a pdf of the following form:

$$f(u) = \begin{cases} A & u \leq 0 \\ u & 0 < u \leq 1 \\ B(2-u) & 1 < u \leq 2 \\ C & u \geq 2 \end{cases}$$

   (a) Find the values of A, B, and C that make f a valid pdf.

   (b) Derive the CDF F(c) corresponding to f.

2. **[Continuous-type random variables II]**
   The pdf of a random variable X is given by:

$$f_X(u) = \begin{cases} a + bu^2, & 0 \leq u \leq 1 \\ 0, & \text{else} \end{cases}$$

   If E[X]=5/8, find a and b.

3. **[Uniform and exponential distribution I]**

   (a) In a new subdivision, along a street of some finite length L, a fire station will be built. If fires will occur at points uniformly chosen on (0, L), where should the station be located so as to minimize the expected distance from the fire? That is, choose a so as to minimize $E[|X - a|]$, when X is uniformly distributed over (0, L)

   (b) Now suppose the street is of infinite length- stretching from point 0 outward to ∞. If the distance of a fire from point 0 is exponentially distributed with rate λ, where should the fire station now be located?

4. **[Uniform and exponential distribution II]**
A factory produced two equal size batches of radios. All the radios look alike, but the lifetime of a radio in the first batch is uniformly distributed from zero to two years, while the lifetime of a radio in the second batch is exponentially distributed with parameter $\lambda = 0.1(\text{years})^{-1}$.

   (a) Suppose Alicia bought a radio and after five years it is still working. What is the conditional probability it will still work for at least three more years?

   (b) Suppose Venkatesh bought a radio and after one year it is still working. What is the conditional probability it will work for at least three more years?

5. **[Poisson process]**
A certain application in a cloud computing system is accessed on average by 15 customers per minute. Find the probability that in a one minute period, three customers access the application in the first ten seconds and two customers access the application in the last fifteen seconds. (Any number could access the system in between these two time intervals.)