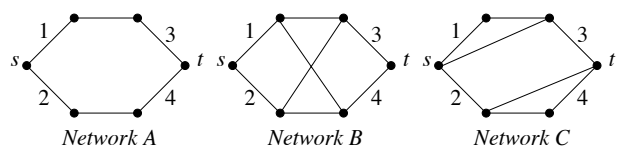
University of Illinois **Fall 2010**

ECE 313: Problem Set 6
Reliability and CDFs

Due:	Wednesday, October 6, at 4 p.m.
Reading:	*ECE 313 Notes* Sections 2.12 & 3.1
Reminder:	Hour Exam I on Monday October 11, 7:00 p.m. – 8:00 p.m. Sections C&D Room 141 Wohlers Hall, Sections X&E Room 114, David Kinley Hall One two-sided 8.5"×11" sheet of notes allowed, with font size no smaller than 10 pt or equivalent handwriting. Bring a picture ID. The exam will cover the reading assignments, lectures, and problems associated with problem sets 1-6. The TAs will lead optional review sessions during the regular lecture times on Friday, October 8.

1. **[Reliability of three $s-t$ networks]**
Consider $s-t$ networks A through C shown.



For each network, suppose that each link fails with probability p, independently of the other links, and suppose network outage occurs if at least one link fails on each path from s to t. Let F denote the event of network outage.

(a) Find $P(F)$ in terms of p for Network A, and give the numerical value of $P(F)$ for $p = 0.001$, accurate to within four significant digits.

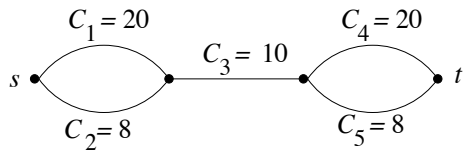
(b) This part aims to find $P(F)$ for Network B, using the law of total probability, based on the partition of the sample space into the events: $D_0, D_1, D_{2,s}, D_{2,d}, D_3, D_4$. Here, D_i is the event that exactly i links from among links $\{1,2,3,4\}$ fail, for $i = 0,1,3,4$; $D_{2,s}$ is the event that exactly two links from among links $\{1,2,3,4\}$ fail and they are on the same side (i.e. either links 1 and 2 fail or links 3 and 4 fail); $D_{2,d}$ is the event that exactly two links from among links $\{1,2,3,4\}$ fail and they are on different sides. Find the probability of each of these events, find the conditional probabilities of F given any one of these events, and finally, find $P(F)$. Express your answers as a function of p, and give the numerical value of $P(F)$ for $p = 0.001$, accurate to within four significant digits.

(c) Find the numerical value of $P(D_{2,d}|F)$ for $p = 0.001$.

(d) Find the limit of the ratio of the outage probability for Network B to the outage probability for Network A, as $p \to 0$. Explain the limit. Is it zero? Hint: For each network, $P(F)$ is a polynomial in p. As $p \to 0$, the term with the smallest power of p dominates.

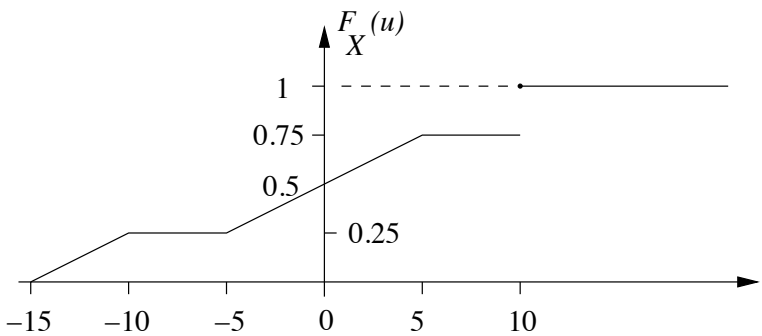
(e) Without doing any detailed calculations, based on the reasoning used in part (d), give the limit of the ratio of the outage probability for Network C to the outage probability for Network A, as $p \to 0$. Explain the limit. Is it zero?

2. **[Distribution of capacity of an $s-t$ flow network]**
Consider the following $s-t$ flow network. The link capacities, in units of some quantity per unit time, are shown for links that do not fail. Suppose each link fails with some probability p and if a link fails it can carry no flow. Let Y denote the $s-t$ capacity of the network.



Find the pmf of Y. Express your answer in terms of p. Te facilitate grading, express each nonzero term of the pmf as a polynomial in p, with terms arranged in increasing powers of p. (Hint: What is $P\{Y > 0\}$?)

3. **[Using a CDF]**
Let X be a random variable with the CDF shown.



Compute the following probabilities:

(a) $P\{X \leq 10\}$
(b) $P\{X \geq -7\}$
(c) $P\{|X| < 10\}$
(d) $P\{X^2 \leq 16\}$

4. **[Recognizing a valid CDF]**
Which of the six plots below show valid CDFs? For each one that is not valid, state a property of CDFs that is violated.