# ECE 313: Problem Set 3
# Conditional probabilities, independence, and the binomial distribution

**Due:** Wednesday September 15 at 4 p.m.
**Reading:** *313 Course Notes* Sections 2.2–2.4.

1. **[Conditional probability]**
   Two fair dice are rolled. What is the conditional probability that at least one lands on six given that the dice land on different numbers?

2. **[Ultimate verdict]**
   Suppose each time a certain defendant is given a jury trial for a particular charge (such as trying to sell a seat in the US Senate), an innocent verdict is given with probability $q_I$, a guilty verdict is given with probability $q_G$, and a mistrial occurs with probability $q_M$, where $q_I, q_G$, and $q_M$ are positive numbers that sum to one. Suppose the prosecutors are determined to get a guilty or innocent verdict, so that after any number of consecutive mistrials, another trial is given. The process ends immediately after the first trial with a guilty or innocent verdict; appeals are not considered. Let $T$ denote the total number of trials required, and let $I$ denote the event that the verdict for the final trial is innocent.

   (a) Find $P(I|T = 1)$. Express your answer in terms of $q_I$ and $q_G$.

   (b) Find the pmf of $T$.

   (c) Find $P(I)$. Express your answer in terms of $q_I$ and $q_G$.

   (d) Compare your answers to parts (a) and (c). For example, is one always larger than the other?

3. **[Independence]**
   Suppose two dice, one orange and one blue, are rolled. Define the following events:
   A: The product of the two numbers that show is 12
   B: The number on the orange die is strictly larger than the number on the blue die.
   C: The sum of the numbers is divisible by four.
   D: The number on the orange die is either 1 or 3.

   (a) List all pairs of events from the set $A, B, C$, and $D$ that are independent.

   (b) List all triplets of events, if any, from the set of $A, B, C$, and $D$, that are mutually independent.

4. **[Binomial distribution I]**
   Five cars start out on a cross-country race. The probability that a car breaks down and drops out of the race is 0.2. Cars break down independently of each other.

   (a) What is the probability that exactly two cars finish the race?

   (b) What is the probability that at most two cars finish the race?

   (c) What is the probability that at least three cars finish the race?

5. **[Binomial distribution II]**
   A New Yorker runs an investment management service that has the stated goal of doubling the value of his clients' investments in a week via day trading. His brochure boasts that, "On average, my clients triple their money in five weeks!" After poring over back issues of the *Wall Street Journal* you learn the truth: at the end of any week, the investments of his clients will have doubled with probability 0.5, and will have decreased by 50% with probability 0.5. Thus, at the end of the first week, an initial investment of \$32 will be worth either \$64 or \$16, each with probability 0.5. Performance in any week is independent of performance during the other weeks. Anxious to apply your new skills in probability theory, you decide to invest \$32, and to let that investment ride for five weeks (in fact, you decide not to even look at the stock prices until the five weeks are over). Let the random variable $X$ denote the value in dollars of your investment at the end of a five week period.

(a) What are the possible values of $X$?

(b) What is the pmf of the random variable $X$?

(c) What is the expected value of $X$? Is the TV commercial accurate?

(d) What is the probability that you will lose money on your investment?