University of Illinois **Fall 2010**

ECE 313: Problem Set 2
Discrete random variables

Due: Wednesday September 8, at 4 p.m..
Reading: ECE 313 notes, Chapter 1 and Sections 2.1-2.2.

1. **[A classification of students in a class]**
   Of 30 students in a class,

   - 12 are not on Facebook
   - 9 are on both Facebook and Twitter
   - 2/3 of students not on Twitter don't have iPads
   - At least one student is neither on Twitter nor on Facebook
   - 3 students on Facebook and having iPads are not on Twitter
   - 2 students are on both Facebook and Twitter and have iPads
   - 2/3 of the students not on Facebook and without iPads are on Twitter

   Find how many students are not on Twitter or Facebook and don't have iPads. Show your work. (Hint: Use a Karnaugh map. Fill in numbers or variables, trying to minimize the number of variables and equations needed.)

2. **[Distribution of number of matches]**
   Suppose four people write their names on slips of paper; the slips of paper are randomly shuffled and then each person gets back one slip of paper; all possibilities of who gets what slip are equally likely. Let X denote the number of people who get back the slip with their own name (i.e. the number of matches).

   (a) Describe a suitable sample space Ω to describe the experiment. How many elements does it have?
   (b) Find the pmf of X.
   (c) Find $E[X]$.
   (d) Find the probability that a given person gets her/his own name. Explain how this question is related to part (c).
   (e) Find $\mathrm{Var}(X)$.

3. **[A problem on sampling without replacement]**
   A bag contains n pairs of shoes in distinct styles and sizes. You pick two shoes at random from the bag. Note that this is sampling *without* replacement.

   (a) What is the probability that you get a pair of shoes?
   (b) What is the probability of getting one left shoe and one right shoe?

   Suppose now that $n \geq 2$ and that you choose 3 shoes at random from the bag.

   (c) What is the probability that you have a pair of shoes among the three that you have picked?
   (d) What is the probability that you picked at least one left shoe and at least one right shoe?

4. **[Mean and standard deviation of a complicated random variable]**
   (Use a spreadsheet program, Matlab, programmable calculator, or your favorite computer language for this problem.) Suppose two fair dice are rolled independently, so the sample space is $\Omega = \{(i,j) : 1 \leq i \leq 6, 1 \leq j \leq 6\}$ and all outcomes are equally likely. Let X be the random variable defined by $X(i,j) = (i-j)^2 - |2i-j|$.

(a) Calculate $E[X]$. (Hint: This is just the average of the 36 values of X.)

(b) Calculate $E[X^2]$. (Hint: This is just the average of the 36 values of X^2.)

(c) Using your answers to (a) and (b), calculate the standard deviation of X. (Note: If you were to apply the STDEV function of the Excel spreadsheet to the list of 36 values of X, you would not get the standard deviation of X, because the STDEV function is for *estimating* the standard deviation from n samples, and it uses what is called the $n-1$ rule. The STDEVP function returns the correct value, and can be used for you to check your answer.)

(d) Find the pmf of X, $p_X(k)$, for $k = 0, 1, 2$. (Note: In principle, you could compute the complete pmf of X and use it to do (a) and (b) above, but the hints for (a) and (b), based on the law of the unconscious statistician (LOTUS), give a much simpler way to do (a) and (b).)

5. **[Mean and standard deviation of two simple random variables]**
Suppose two fair dice are rolled independently, so the sample space is $\Omega = \{(i, j) : 1 \leq i \leq 6\}$ and all outcomes are equally likely. Let X be the number showing on the first die, $X(i, j) = i$, and let Y be the random variable defined by $Y(i, j) = \min\{i, j\}$.

(a) Derive the pmf of X and sketch it.

(b) Find the mean $E[X]$ and standard deviation, σ_X, of X. Correct numerical answers are fine, but show your work.

(c) Derive the pmf of Y and sketch it.

(d) Find the mean $E[Y]$ and standard deviation, σ_Y, of Y. Correct numerical answers are fine, but show your work. (Hint: It may be helpful to reuse the spreadsheet or program you used for Problem 4.)

(e) Which is larger, σ_X or σ_Y? Is that consistent with your sketches of the pmfs?

(f) The random variable X takes values in the set $\{1, 2, 3, 4, 5, 6\}$. Specify another pmf on the same set which has a larger standard deviation than X.