University of Illinois **Fall 2010**

ECE 313: Problem Set 1
Axioms of probability and calculating the sizes of sets

Due: Wednesday, September 1 at 4 p.m.
Reading: *ECE 313 Course Notes,* Sections 1.1–1.4.

1. [**Defining a set of outcomes**]
Suppose four teams, numbered one through four, play a single-elimination tournament, consisting of three games. Two teams play each game and one of them wins; ties do not occur. Teams one and two play each other in the first game; teams three and four play each other in the second game; the winner of the first game plays the winner of the second game in the third game.

   (a) Define a set Ω so that the elements of Ω correspond to the possible scenarios of the tournament. An element of Ω should specify the entire sequence of outcomes of the games. Explain how the elements of your set correspond to the possible scenarios. (This is an exercise in coming up with good notation.)

   (b) How may possible scenarios are there?

2. [**Possible probability assignments**]
Suppose A and B are events for some probability space such that $P(AB) = 0.3$ and $P(A \cup B) = 0.6$. Find the set of possible values of the pair $(P(A), P(B))$ and sketch this set, as a subset of the plane. Hint: It might help to try filling in the probabilities in a Karnaugh map.

3. [**Grouping students into teams**]
Suppose ten students in a class are to be grouped into teams.

   (a) If each team has two students, how many ways are there to form teams? (The ordering of students within teams does not matter, and the ordering of the teams does not matter.)

   (b) If each team has either two or three students, how many ways are there to form teams?

4. [**A Karnaugh map for three events**]
Let an experiment consist of rolling two fair dice, and define the following three events about the numbers showing: $A =$"sum is even," $B =$"sum is a multiple of three," and $C =$"the number showing on the first die is (strictly) less than the number showing on the second die."

   (a) Display the outcomes in a three-event Karnaugh map, as in Example 1.4.2.

   (b) Find $P((A \cup B)C)$.

5. [**Two more poker hands**]
Suppose five cards are drawn from a standard 52 card deck of playing cards, as described in Example 1.4.3, with all possibilities being equally likely.

   (a) *FLUSH* is the event that all five cards have the same suit. Find $P(FLUSH)$.

   (b) *FOUR OF A KIND* is the event that four of the five cards have the same number. Find $P(FOUR\ OF\ A\ KIND)$.