ECE 313: Hour Exam I

Monday October 11, 2010
7:00 p.m. — 8:00 p.m.
Room 141 Wohlers Hall & Room 114 David Kinley Hall

1. (a) \( P(ABC^c) = 0.01. \)
(b) \( P(AB \cup BC \cup AC) = 0.12. \)
(c) \( P(A \cup B \cup C)^c = 0.68. \)
(d) \( P(AB \cup BC) = 0.11. \)
(e) \( P(ABC^c \cup A^cBC) = 0.1. \)

2. (a) Let \( T \) represent the event that the exam committee is tough.
\[
P(T) = \frac{6 \binom{4}{2} + \binom{4}{3}}{\binom{10}{3}} = \frac{6 \cdot 6 + 4 \cdot 4}{3 \cdot 4 \cdot 5} = \frac{40}{120} = \frac{1}{3}
\]

(b) Let \( A \) denote the event a petition is approved. By the law of total probability,
\[
P(A) = P(A|T)P(T) + P(A|T^c)P(T^c) = (0.5)(1/3) + (0.8)(2/3) = 0.7
\]

(c) The number of tries until success has the geometric distribution with parameter \( p = 3/4 \), so the expected number of tries is \( 1/p = 4/3 \).

(d) The probability the petition is not eventually approved is the probability it is not approved by each of three committees, which is \((1 - p)^3 = 27/64\). Thus, a petition is eventually approved with probability \( 37/64 \).

3. (a)
\[
P(\text{exactly one}|\text{at least one}) = \frac{P\{\text{(exactly one) \cap (at least one)}\}}{P\{\text{at least one}\}}
\]
\[
= \frac{P\{\text{exactly one}\}}{P\{\text{at least one}\}}
\]
\[
= \frac{\binom{4}{1}(0.5)(0.5)^3}{1 - (0.5)^4} = \frac{4 \cdot 0.125}{0.625} = \frac{4}{15}
\]

(b) There are 18 flights per day, so 5/18 are American, 4/18 are AirTrans, and 9/18 are Delta. These fractions give the pmf for the airline associated with a random flight. Thus, by the law of total probability, the probability a randomly selected flight departs late is
\[
P\{\text{flight departs late}\} = \frac{5}{18} \cdot 0.2 + \frac{4}{18} \cdot 0.05 + \frac{9}{18} \cdot 0.1 = \frac{2.1}{18} = \frac{7}{60}.
\]

(c)
\[
P(\text{flight is American}|\text{flight departs late}) = \frac{P\{\text{flight is American and departs late}\}}{P\{\text{flight departs late}\}}
\]
\[
= \frac{\frac{5}{18} \cdot 0.2}{\frac{7}{60}} = \frac{1}{7} \cdot \frac{30}{7} = \frac{10}{21}.
\]
4. (a) The likelihood that $X = 4$ is $\frac{\lambda^4 e^{-\lambda}}{4!}$, and $\hat{\lambda}_{ML}(4)$ is the value of $\lambda$ that maximizes the likelihood, or equivalently, maximizes $\lambda^4 e^{-\lambda}$. Since $(\lambda^4 e^{-\lambda})' = (4\lambda^3 - \lambda^4)e^{-\lambda} = (4 - \lambda)\lambda^3 e^{-\lambda}$, the likelihood is increasing in $\lambda$ for $\lambda < 4$ and decreasing in $\lambda$ for $\lambda > 4$; it is maximized at $\lambda = 4$. Thus, $\hat{\lambda}_{ML}(4) = 4$.

(b) This is the same as the probability exactly two even numbers show. Each die shows an even number with probability 0.5, so the number of even numbers showing, $X$, has the binomial distribution with parameter $p = 0.5$. Therefore, $P\{X = 2\} = \binom{4}{2}(0.5)^2(0.5)^2 = \frac{6}{16} = \frac{3}{8}$.

(c) Each roll does not produce two even and two odd numbers with probability $5/8$, and for that to happen on three rolls has probability $(5/8)^3 = \frac{125}{512}$. 