1. (a) $P\{T \geq t\} = F^c_T(t) = e^{-\lambda t} = e^{-(\ln 2)t} = 2^{-t}$.
   
   (b) $P(T \leq 1 | T \leq 2) = \frac{P(T \leq 1, T \leq 2)}{P(T \leq 2)} = \frac{P(T \leq 1)}{P(T \leq 2)} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$.

Suppose $X$ has the pdf shown:

(c) The area under the pdf is one half the base of the triangle times the height, equal to $c$. So $c = 1$.

(d) To get mean zero, let $a = E[X] = 1$. Subtracting $a$ causes the pdf to be centered at zero. And $b = \sigma_X = \sqrt{\frac{1}{6}}$. So $\tilde{X} = \sqrt{6}(X - 1)$. The support of $\tilde{X}$ is thus the interval $(-\sqrt{6}, \sqrt{6})$, and the shape of the pdf is the same triangular shape that $f_X$ has. So the pdf of $\tilde{X}$ is the following:

2. (a) Since $X$ ranges over the interval $[0, 3]$, $Y$ ranges over the interval $[0, 4]$.
   
   (b) The expression for $F_Y(c)$ is qualitatively different for $0 \leq c \leq 1$ and $1 \leq c \leq 4$, as seen in the following sketch:

In each case, $F_Y(c)$ is equal to one third the length of the shaded interval. For $0 \leq c \leq 1$,

$$F_Y(c) = P\{(X - 1)^2 \leq c\} = P\{1 - \sqrt{c} \leq X \leq 1 + \sqrt{c}\} = \frac{2\sqrt{c}}{3}.$$
For $1 \leq c \leq 4$,
\[
F_Y(c) = P((X - 1)^2 \leq c) = P\{0 \leq X \leq 1 + \sqrt{c}\} = \frac{1 + \sqrt{c}}{3}.
\]
Combining these observations yields:
\[
F_Y(c) = \begin{cases} 
0 & c < 0 \\
\frac{2\sqrt{c}}{3} & 0 \leq c < 1 \\
\frac{1+\sqrt{c}}{3} & 1 \leq c < 4 \\
\frac{1}{3} & c \geq 4
\end{cases}
\]
(c) By LOTUS,
\[
E[Y] = E[(X - 1)^2] = \int_0^3 (u - 1)^2 \frac{1}{3} du = 1
\]
3. (a) From the figure, the pdf for $H_1$ is smaller than the pdf for $H_0$ precisely when $b < |u| < a$. Thus, the ML rule is given by:
\[
\hat{H} = \begin{cases} 
H_0 & b < |X| < a \\
H_1 & \text{otherwise}
\end{cases}
\]
(b) These probabilities are calculated as follows:
\[
p_{false\; alarm} = \frac{2b}{2a} = \frac{b}{a}
\]
\[
p_{miss} = 2(\Phi(a) - \Phi(b)) = 2(Q(b) - Q(a))
\]
4. (a) No. For example, the support is not a product set because $(0.2, 0.8)$ and $(0.8, 0.2)$ are in the support of $f_{X,Y}$ but $(0.8, 0.8)$ is not.
(b) $P\{X \leq Y\} = P\{Y \leq X\}$ by symmetry and $P\{X \leq Y\} + P\{Y \leq X\} = 1$ (because $P\{X = Y\} = 0$) so $P\{X \leq Y\} = 0.5$.
(c) The set $\{X \leq 0.5, Y \leq 0.5\}$ intersected with the support of $f_{X,Y}$ is the square region shown:
Thus, \( P\{X \leq 0.5, Y \leq 0.5\} = \int_0^{0.5} \int_0^{0.5} 8uv \, du \, dv = \frac{1}{8}. \)

(d) By LOTUS, \( E\left[\frac{1}{XY}\right] = \int \int \frac{1}{uv} f_{X,Y}(u,v) \, du \, dv = \int \int_{\text{support}} 8 \, du \, dv = 2\pi. \)