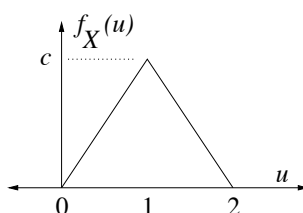# ECE 313: Hour Exam II

Monday November 15, 2010
7:00 p.m. — 8:00 p.m.
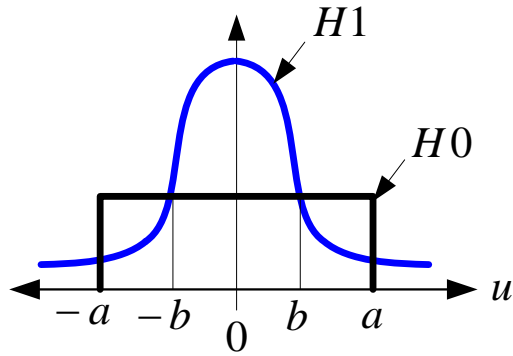124 Burrill Hall & Room 100 Noyes Lab

1. [**25 points**] Let T be an exponentially distributed random variable with parameter $\lambda = \ln 2$.

   (a) [**6 points**] Find the simplest expression possible for $P\{T \geq t\}$ as a function of t.
   (b) [**6 points**] Find $P(T \leq 1 | T \leq 2)$.

   Suppose X has the pdf shown:

   (c) [**5 points**] Find the constant c.

   (d) [**8 points**] Let $\widetilde{X}$ denote the standardized version of X. Thus, $\widetilde{X} = \frac{X-a}{b}$ for some constants a and b so that $\widetilde{X}$ has mean zero and variance one. Carefully **sketch** the pdf of $\widetilde{X}$. Be sure to indicate both the horizontal and vertical scales of your sketch by labeling at least one nonzero point on each of the axes. (Hint: $\text{Var}(X) = \frac{1}{6}$.)

2. [**25 points**] Suppose X is uniformly distributed over the interval $[0, 3]$, and $Y = (X - 1)^2$.

   (a) [**3 points**] What is the set of possible values of Y?
   (b) [**13 points**] Find the CDF, $F_Y(c)$ for $-\infty < c < \infty$.
   (c) [**9 points**] Find $E[Y]$.

3. [**25 points**] An observation X is drawn from a standard normal distribution (i.e. $N(0,1)$) if hypothesis H_1 is true and from a uniform distribution with support $[-a, a]$ if hypothesis H_0 is true. As shown in the figure below (under part (b)), the pdfs of the two distributions are equal when $|u| = b$.

   (a) [**9 points**] Express the maximum likelihood (ML) decision rule in a simple way, in terms of the observation X and the constants a and b.

   (b) [**8 points**] Shade and label the regions in the figure below such that the area of one of the regions is $p_{false\ alarm}$ and the area of the other region is p_{miss}.

(c) [**8 points**] Express $p_{false\ alarm}$ and p_{miss} for the ML decision rule in terms of the constants a, b, and the Φ function or Q function with positive arguments.

4. [**25 points**] Let X and Y have joint pdf $f_{X,Y}(u,v) = \begin{cases} 8uv & \text{if } u \geq 0, v \geq 0, \ u^2 + v^2 \leq 1 \\ 0 & \text{else.} \end{cases}$

    (a) [**6 points**] Are X and Y independent random variables? Justify your answer.

    (b) [**5 points**] Find $P\{X \leq Y\}$.

    (c) [**7 points**] Find $P\{X \leq 0.5, Y \leq 0.5\}$.

    (d) [**7 points**] Find $E\left[\frac{1}{XY}\right]$.