University of Illinois **Fall 2009**

ECE 313: Problem Set 2
Axioms of Probability; Countably Infinite Sample Spaces

Due: Wednesday September 9 at 4 p.m..
Reading: *Ross* Chapter 1, Sections 1-4; Chapter 2, Sections 1-5
Powerpoint Lecture Slides, Sets 4-7

1. [**Maximum and minimum values for probabilities**]
   A, B and C are events with probabilities 0.6, 0.2, and 0.7 respectively.
   (a) What are the largest possible values of $P(A \cup B)$ and $(A \cup C)$?
   (b) What are the smallest possible values of $P(A \cup B)$ and $(A \cup C)$?
   (c) What are the largest possible values of $P(A \cap B)$ and $(A \cap C)$?
   (d) What are the smallest possible values of $P(A \cap B)$ and $(A \cap C)$?

2. [**Unions and Intersections**]
   Find the value of $P(A \cap (B \cup C))$ in each of the following three cases. If there is not enough information to calculate the desired probability, then state so.
   (a) $A, B,$ and C are mutually exclusive events.
   (b) $P(C) = 0.35$, $P(A \cap B) = 0.2$, $P(A \cap C) = 0.3$, $P(B) = 0.25$, $P(B \cup C) = 0.6$.
   (c) $P(A) = P(B) = P(C) = \frac{1}{3}$.

3. [**Countably infinite sample space**]
   A fair die is rolled once. Let $f_1 \in \{1, 2, 3, 4, 5, 6\}$ denote the outcome. The die is then rolled repeatedly till an outcome f_2 that is *different* from f_1 occurs.
   (a) Find the probability that f_1 is even.
   (b) Find the probability that both f_1 and f_2 are even.
   (c) Find the probability that $f_1 + f_2 \leq 7$.

4. [**Maximizing your chances**]
   In an unusual tennis match, you play one set each against three opponents (1, 2, 3) in that order, and win the match if you win *two consecutive sets*. The probabilities of winning a set against these three opponents are p_1, p_2, p_3, respectively.
   (a) Express the probability of winning the match in terms of p_1, p_2, p_3.
   (b) Suppose you can *choose* the order in which you play your opponents. Show that choosing to play the easiest opponent in the second set will maximize your chance of winning the tournament.

5. [**Countably infinite sample spaces**]
   $\Omega = \{0, 1, 2, \ldots\}$ is a countably infinite sample space with $P(n) = \dfrac{(\log_e 2)^n}{2(n!)}$ for all $n \geq 0$. Remember that $0! = 1$.
   (a) Show that $P(\Omega) = 1$ for this probability assignment.
   (b) Prove that the probability that the outcome is an even number is $5/8$. Remember that 0 is an even number.

6. [**A Coin Tossing Game**]
   Bob & Carol & Ted & Alice take turns (in that order) tossing a coin with $P(H) = p$, $0 < p < 1$. The first one to toss a Head wins the game.
   (a) Calculate their *win probabilities* $P(B), P(C), P(T)$, and $P(A)$.
   (b) Show that $P(B) > P(C) > P(T) > P(A)$.