

3. [Extrema of functions]

- (a) Find all the extreme values (maxima and minima) of $x^{25}(1.00001)^{-x}$ in the interval $(0, \infty)$.
(b) Find the maximum value and the minimum value of $f(x) = \exp(-|x|)$ on the interval $[-1, 2]$.

4. [Some definite integrals]

Find the values of the following **definite** integrals:

(a) $\int_{-2}^1 |x| dx$; (b) $\int_0^1 x(1-x^2)^{11} dx$; (c) $\int_0^1 x^2 \exp(-x) dx$; (d) $\int_{-10}^{10} x^3 \exp(-x^2/2) dx$.

5. [Derivatives and integrals]

Let $\frac{d}{dx}f(x) = g(x)$, $-\infty < x < \infty$ and let C denote an arbitrary constant. Which of the following statements is true for all x ?

- (a) $\frac{d}{dx}f(-x) = -g(-x)$ (b) $\frac{d}{dx}f(x^2/2) = xg(x^2/2)$ (c) $\frac{d}{dx} \exp(f(x^2)) = g(x^2) \exp(f(x^2))$
(d) $\int g(-x) dx = f(-x) + C$ (e) $\int g(x^2/2) dx = \frac{f(x^2/2)}{x} + C$ (f) $\int \frac{g(x)}{f(x)} dx = \ln(f(x)) + C$

6. [Double integrals]

Evaluate the following **definite** two-dimensional integrals over the specified domains of integration:

- (a) $f(x, y) = \min(x, y)$, over the region $\{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 1\}$.
(b) $f(x, y) = \exp(-\frac{1}{2}(x^2 + y^2))$ over the region $\{(x, y) : x^2 + y^2 > 4\}$.

Hint: change to polar coordinates; then, before tackling the resulting integral, compute the derivative of $\exp(-r^2/2)$ and stare at it for a while!