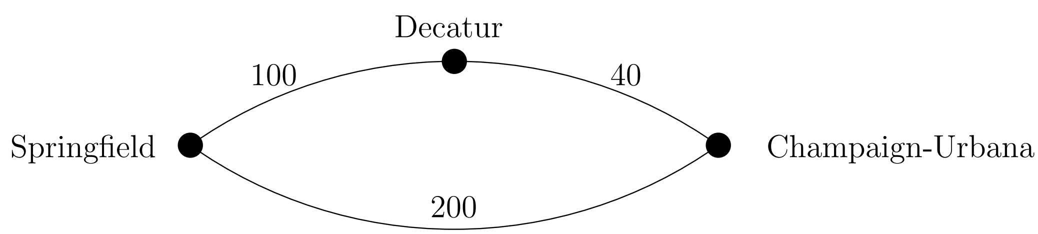
University of Illinois Fall 2009

ECE 313: Hour Exam II

Monday November 16, 2009
7:00 p.m. — 8:00 p.m.
100 Materials Science and Engineering Building

1. **[15 points]** Messages can be transmitted between Champaign-Urbana and Springfield either directly or via Decatur on communication links whose capacities are indicated on the diagram shown below. Assume that the three links fail independently with equal probability $\frac{1}{2}$. Let $\mathbb{X}$ denote the communication capacity between Champaign-Urbana and Springfield.



   (a) **[5 points]** What values can $\mathbb{X}$ take on?

   (b) **[10 points]** Find the expected value of $\mathbb{X}$.

2. **[10 points]** Let $\mathbb{X}$ denote a *Gaussian* random variable with mean 2 and variance 25.

   Use the table of values of the unit Gaussian CDF $\Phi(\cdot)$ on the last page of this exam booklet to find the numerical value of $P\{|\mathbb{X} - 4| > 3\}$.

3. **[15 points]** Let $\mathbb{X}$ denote a continuous random variable with zero mean and variance 1, that is, $\mathsf{E}[\mathbb{X}] = 0$ and $\mathsf{var}(\mathbb{X}) = 1$.

   (a) **[10 points]** Find the mean and variance of $\mathbb{Y} = 2\mathbb{X} + 3$.

   (b) **[5 points]** Let $\mathbb{Z} = |\mathbb{X}|$.
   Mark TRUE or FALSE by checking one box below. You need not justify your answer.

TRUE	FALSE	
☐	☐	The variance of $\mathbb{Z}$ is greater than 1, that is $\mathsf{var}(\mathbb{Z}) > 1$.

4. **[10 points]** Let $\mathbb{X}$ denote an *exponential* random variable with mean 2.

   (a) **[5 points]** Find $P\{\mathbb{X} > 1\}$.

   (b) **[5 points]** Calculate $P\{\mathbb{X} > 3 \mid \mathbb{X} > 2\}$.

5. **[25 points]** $\mathbb{X}$ denotes a *uniform* random variable with mean 1 and variance 3.

   (a) **[10 points]** Find $P\{\mathbb{X} < 0\}$.

   (b) **[15 points]** Find the pdf of $\mathbb{Y} = |\mathbb{X}|$. In order to receive full credit, you must specify the value of $f_{\mathbb{Y}}(v)$ for all $v, -\infty < v < \infty$.

6. **[25 points]** Consider the following binary hypothesis testing problem.

   If hypothesis H_0 is true, the continuous random variable $\mathbb{X}$ is uniformly distributed on $(-2, 2)$, while if hypothesis H_1 is true, the pdf of $\mathbb{X}$ is $f_1(u) = \begin{cases} \frac{|u|}{4}, & |u| < 2, \\ 0, & \text{otherwise.} \end{cases}$

(a) **[5 points]** Find the decision region Γ_0 for the *maximum-likelihood* decision rule. Remember that Γ_0 is the set of all real numbers such that if $\mathbb{X} \in \Gamma_0$, the decision is that H_0 is the true hypothesis.

(b) **[10 points]** Find the probability of false alarm P_{FA} and the probability of missed detection P_{MD} for the maximum-likelihood decision rule.

(c) **[10 points]** Suppose that the hypotheses have *a priori* probabilities $\pi_0 = 1/3$ and $\pi_1 = 2/3$. Find the decision region Γ_0 for the maximum *a posteriori* probability (MAP) decision rule (also known as the minimum-error-probability or Bayesian decision rule).