University of Illinois Fall 2009

ECE 313: Hour Exam I

Monday October 12, 2009
7:00 p.m. — 8:00 p.m.
100 Noyes Laboratory

1. **[15 points]** Let A, B, and C denote three events defined on a sample space Ω, and suppose that $P(A) = 0.6, P(B) = P(C) = 0.3$, and $P(B^c \cap C) = P(A \cap B^c \cap C^c) = 0.2$.

   (a) **[5 points]** Find $P(B \cap C)$.

   (b) **[5 points]** Find $P(B \cap C^c)$.

   (c) **[5 points]** Find $P((A \cup B \cup C)^c)$.

2. **[10 points]** A and B are events defined on a sample space Ω. Assume $P(A), P(B) > 0$. Mark each of the two statements below as TRUE or FALSE. *No justification is needed.*

TRUE	FALSE	
☐	☐	If $P(A \mid B) = P(B \mid A)$, then $P(A) = P(B)$.
☐	☐	$P(A \mid B)P(B) + P(A^c \mid B)P(B) = P(B)$.

3. **[30 points]** *Especially in this problem, you must provide sufficient explanation to justify your numerical answers.*

   A fair coin is tossed repeatedly until a Head occurs. N denotes the number of tosses.

   (a) **[5 points]** What is the expected value of N?

   (b) **[5 points]** Find the *numerical value* of $P\{N > 5\}$.

   (c) **[10 points]** Given that the event $P\{N > 5\}$ occurred, what is the expected value of N?

   (d) **[10 points]** Find the *numerical value* of $\mathsf{E}[\cos(\pi N)]$.

4. **[20 points]** A fair coin is tossed 10 times.

   Calculate the probability that the first 5 tosses are all Tails given that a total of 8 Tails occurred on the 10 tosses.

5. **[25 points]** Dilbert has 3 coins in his pocket, 2 of which are fair coins while the third is a biased coin with $P(H) = p \neq \frac{1}{2}$. The probability that a coin chosen at random from his pocket will land Tails is $\frac{7}{12}$.

   (a) **[10 points]** What is the value of p?

   (b) **[15 points]** Dilbert picks two coins at random from his pocket, tosses each coin once, and observes a Head and a Tail. What is the conditional probability that both coins are fair?