ECE 313: Hour Exam II

Monday November 17, 2008
7:00 p.m. — 8:00 p.m.
Room 1404 Siebel Center

1. [20 points] Consider an experiment in which a **fair tetrahedral** (that is, **four-sided** die is rolled. The faces of the die are marked 1, 2, 3, and 4, and the outcome is defined as the number on the bottom face when the die stops rolling. Thus, the sample space is $\Omega = \{1, 2, 3, 4\}$. Let $E_2 = \{1, 2\}, E_3 = \{1, 3\}$, and $E_4 = \{1, 4\}$ be events.

   (a) [6 points] Are the events E_2 and E_3 (mutually) **independent** events?
   (b) [7 points] Are the events E_2, E_3, and E_4 **pairwise independent**?
   (c) [7 points] Are the three events E_2, E_3, and E_4 **mutually independent?**

2. [30 points]

   (a) [12 points] For what value(s) of α is $f(u) = \begin{cases} (1 + \alpha) - 2\alpha u, & 0 \le u \le 1, \\ 0, & \text{otherwise}, \end{cases}$

   a valid probability density function (pdf)?

   (b) [18 points] Let $\mathcal{X}$ denote a continuous random variable with probability density function (pdf) $f_{\mathcal{X}}(u) = \begin{cases} a + bu, & 0 \le u \le 1, \\ 0, & \text{otherwise}. \end{cases}$

   Given that $\mathsf{E}[\mathcal{X}] = \dfrac{2}{3}$, what is $P\left\{\mathcal{X} < \dfrac{1}{2}\right\}$?

3. [20 points] Let $\mathcal{X}$ denote a **Gaussian** random variable with mean 10 and variance 100.

   Find the value of $P\{\mathcal{X} > 31.7 | \mathcal{X} > 29.6\}$. A table of values of $\Phi(x)$, the standard Gaussian CDF, is attached to this examination.

4. [30 points] Let $\mathcal{X}$ denote an **exponential** random variable with parameter $\lambda = 1$. Find the probability density function $f_{\mathcal{Y}}(v)$ of the random variable $\mathcal{Y} = \sqrt{\mathcal{X}}$.

   To obtain full credit, you must specify the value of $f_{\mathcal{Y}}(v)$ for all v, $-\infty < v < \infty$.