1. [40 points] A store in the Paynless Shoe Sores chain has 13 different pairs of shoes in a barrel at the door. Suppose that two shoes are picked at random from the barrel. Note that this is sampling without replacement.

(a) [18 points] What is the probability of getting a matching pair of shoes? What is the probability of getting two left shoes? What is the probability of getting a left shoe and a right shoe but not a matching pair of shoes?

(b) [6 points] Two more shoes are picked at random from the barrel without replacing the first two shoes picked back in the barrel. What is the probability that these two shoes are a matching pair of shoes?

(c) [8 points] What is the probability that there is at least one matching pair of shoes in the four that have been picked?

(d) [8 points] Given that there are no matching pairs of shoes in the four that have been picked, what is the (conditional) probability that all four shoes are left shoes?

2. [20 points] Dilbert has three coins in his pocket, two of which are fair, and one of which is biased with \( P(\text{Heads}) = \frac{3}{4} \).

(a) [5 points] If Dilbert picks two coins out of his pocket what is the probability that he did not pick the biased coin?

(b) [15 points] If Dilbert picks two coins out of his pocket, tosses each one once, and observes a Head and a Tail, what is the (conditional) probability that he did not pick the biased coin?

3. [25 points] Each box of Cornies, the breakfast of silver medalists, contains a picture of either Britney Spears or Paris Hilton. The purchase of each box of Cornies can be regarded as an independent trial of an experiment on which events \( S \) and \( H \) occur with probabilities \( \frac{1}{4} \) and \( \frac{3}{4} \) respectively.

(a) [20 points] Let \( X \) denote the number of boxes of Cornies purchased till the experimenter has acquired at least one picture of each woman. What is \( P\{X = k\} \) for \( k \geq 2 \)? What is \( E[X] \)?

(b) [5 points] Let \( Y \) denote the number of boxes of Cornies purchased till the experimenter has acquired at least two pictures of each woman. What is \( P\{Y = 4\} \)?

4. [15 points]

(a) [9 points] If \( X \) is a binomial random variable with parameters \( (4, \frac{1}{3}) \), what are the mean and variance of the random variable \( 2 + 3X \)?

(b) [6 points] Let \( Y \) be a geometric random variable with parameter \( p \) where the value of \( p \) is unknown. It is observed that \( \{Y = k\} \). What is the maximum likelihood estimate \( \hat{p}_{\text{ML}} \) of the parameter \( p \)?