ECE 413: Probability with Engineering Applications

Spring 2007
Exam I

Monday, February 26, 2007

Name: ____________________________________________________________

• You have 60 minutes for this exam. The exam is closed book and closed note, except you may consult both sides of one 8.5" × 11" sheet of notes in ten point font size or larger, or equivalent handwriting size.

• Calculators, laptop computers, Palm Pilots, two-way e-mail pagers, etc. may not be used.

• Write your answers in the spaces provided.

• Please show all of your work. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page.

Score:

1. _______ (36 pts.)
2. _______ (16 pts.)
3. _______ (12 pts.)
4. _______ (14 pts.)
5. _______ (22 pts.)

Total: _______(100 pts.)
Problem 1 (36 points) An experiment consists of rolling three fair dice. The rolls are independent trials. Let $A$ be the event that the numbers showing on the three dice are all even, and let $B$ be the event that the numbers showing on the three dice are different from each other.

(a) Express the set of possible outcomes $\Omega$ using mathematical set notation.

(b) Find $P(A)$.

(c) Find $P(B)$.

(d) Find $P(AB)$.

(e) Find $P(A \cup B)$.

(f) Sketch and carefully label the probability mass function of $X$, where $X$ denotes the number of distinct numbers showing on the dice.
Problem 2 (16 points) Let $X$ be a random variable with mean 4 and variance 16.

(a) Find the numerical value of $E[X^2]$. Remember to explain your reasoning.

(b) Find the numerical value of $E[(X + 2)(X + 3)]$. Show your work.
**Problem 3** (12 points) Suppose that a random variable $X$ has the pmf

$$p_X(k) = \begin{cases} 
(k - 1)p^2(1 - p)^{k-2} & k = 2, 3, \ldots \\
0 & \text{else}
\end{cases}$$

where $p$ is an unknown parameter with $0 < p < 1$. (i.e., $X$ has the negative binomial distribution with parameters $p$ and $r = 2$.) Suppose it is observed that $X = 14$. What is the maximum likelihood estimate of $p$? Show your work!

**Problem 4** (14 points) Let $A$ and $B$ denote events such that $P(A) = 0.6, P(B) = 0.5$, and $P(A \mid B) = 0.4$.

(a) Find $P(A \cup B)$.

(b) Find $P(B^c \mid A^c)$. 
Problem 5 (22 points) Consider repeated independent tosses of a biased coin with $P(\text{Heads}) = p$, and let $X$ denote the number of tosses required to observe both one Head and one Tail.

(a) What is the minimum possible value of $X$?

(b) Find the probability mass function of $X$.

(c) Find the expected value of $X$. (Find a closed form answer, with no infinite sum.)