University of Illinois | **Problem Set #4** | **ECE 413 Fall 2004**

Assigned: Wednesday, September 22
Due: Wednesday, September 29

Reading Assignment: Note that this is the LAST problem set before the First Hour Exam. This problem set covers the material in Chapter 4, Sections 4.7 – 4.8 (pp. 149 – 168), and Chapter 3, all Sections (pp. 64 – 104) of Ross. You are encouraged to attempt the self-test problems $\#4, 5, 8, 9, 11$ and 15 on pp. 119 – 121, and $\#13, 14$ on p. 185 of Ross; their solutions can be found in Appendix B.

The problems you will turn in, and which will be graded are the following:

23. Problem 60, p. 111, Ross.

24. Theoretical Exercise 29, p. 183, Ross. **Hint:** Solve Theoretical Exercise 28(a) first.

25. This problem has two *unrelated* parts.

    (a) $P(A|B) = 0.3, P(A^c|B^c) = 0.4$, and $P(B) = 0.7$, find $P(A|B^c), P(A)$, and $P(B|A)$.

    (b) $P(G) = P(H) = 2/3$, show that $P(G|H) \geq 1/2$.

26. Let $\mathbf{X}$ denote a *negative* binomial (or Pascal) random variable with parameters (r, p). Then, $\mathbf{X}$ counts the number of trials required to observe r successes where the probability of success on any trial is p. Given that $\mathbf{X} = n$, what is the conditional probability that the i-th trial resulted in a success? To avoid trivialities, assume $n > r$ and also that $n > i$.

27. Suppose that 105 passengers hold reservation for a 100-seat flight from Chicago to Champaign. Then number of passengers showing up for the flight can be modeled as a binomial random variable $\mathbf{X}$ with parameters $(105, 0.9)$.

    (a) Find the probability that all passengers who show up get seats, i.e. find $P\{\mathbf{X} \leq 100\}$.

    (b) Explain why the number of no-shows can be modeled as a Poisson random variable $\mathbf{Y}$, and compute the value of the parameter λ.

    (c) Compute the probability that all passengers who show up get seats based on this Poisson model, i.e. find $P\{\mathbf{Y} \geq 5\}$, and compare to the "more exact" answer of part (a).

28. On a TV game show you are shown three curtains. One curtain conceals a valuable prize, while the other two conceal junk. All three curtains are equally likely to conceal the prize. The host of the show offers you the following deal: pick a curtain, and you can have whatever is behind it. When you pick a curtain, instead of giving you your just deserts, the host (who knows where the prize is) opens one of the remaining curtains to show you that there is junk behind it, and offers the following *new* deal: you can either stick with your original choice, or switch to the remaining (unopened) curtain. Should you switch, or stick with your original choice?

29. Theoretical Exercise 14, p. 181, Ross.

30. The Senate of a certain country has 100 members consisting of 43 Conservative Republicans, 21 Conservative Democrats, 12 Liberal Republicans, and 24 Liberal Democrats. Before each vote, the groups caucus separately. Each group decides *independently* of the other groups whether to support or oppose the motion. *All* members of the group then vote in accordance with the caucus decision.

(a) Let A, B, C, and D respectively denote the events that the four groups vote for a spending plan that will lead to a 50% increase in the DoD budget over the next two years. Suppose that the probabilitiess of theses independent events are $P(A) = 0.9, P(B) = 0.6, P(C) = 0.5$, and $P(D) = 0.2$. What is the probability that the bill passes?

(b) The President vetoes the bill as a budget-breaker. Let E, F, G, and H respectively denote the independent events that the four groups support the motion to override the veto. If these events have probabilities $P(E) = 0.99, P(F) = 0.4, P(G) = 0.6$, and $P(H) = 0.1$, what is the probability that the motion to override the veto passes?

Note: A simple majority (51 or more votes) is required to pass a bill, and a two-thirds majority (67 or more votes) to override a veto.