ECE 313 **Problem Set # 3** **Fall 2002**
Assigned: 9/11/02 **Due: 9/18/02**

Conditional probabilities and Bayes formula

Assigned reading: *Ross* Sections 3.1–3.3 and first five pages of Section 3.4.
Noncredit exercises: Chapter 3, problems 1,2,5,10,12,16,31,38,39,44

1. Randomly selected die
There are three dice in a bag. One has one red face, another has two red faces, and the third has three red faces. One of the dice is drawn at random from the bag, each die having an equal chance of being drawn. The selected die is repeatedly rolled. (a) What is the probability that red comes up on the first roll?
(b) Given that red comes up on the first roll, what is the conditional probability that red comes up on the second roll?
(c) Given that red comes up on the first three rolls, what is the conditional probability that the selected die has red on three faces?

2. Ternary channel
A communication system transmits three signals, s_1, s_2, or s_3, with equal probabilities. The reception is corrupted by noise, causing the transmission to be changed according to the following table of conditional probabilities:

		Receive, j		
		s_1	s_2	s_3
Send, i	s_1	0.8	0.1	0.1
	s_2	0.05	0.9	0.05
	s_3	0.02	0.08	0.9

The entries list the probability of s_j received, given that s_i is sent. For example, if s_1 is sent, the conditional probability of receiving s_3 is 0.1. (a) Compute the probabilities that s_1, s_2, s_3 are received.
(b) Compute the probabilities $P[s_i \text{ sent} \mid s_j \text{ received}]$ for $i, j = 1, 2, 3$.

3. Matching problem
A router receives 8 packets that it knows need to be routed to eight stations A, B, C, D, E, F, G and H, each station receiving exactly one packet. Due to a lightning strike at the router site, it cannot parse the header information in a packet and hence it randomly assigns the packets to stations. So each station receives one packet, but all possibilities for the assignment are equally likely.
(a) Find the probability that the router gets at least 6 packets routed correctly?
(b) Given that Station A does *not* receive the correct packet, what is the probability that *none* of the stations receive their designated packets?

4. Frantic search
At the end of each day Professor Plum puts her glasses in her drawer with probability .90, leaves them on the table with probability .06, leaves them in her briefcase with probability 0.03, and she actually leaves them at the office with probability 0.01. The next morning she has no recollection of where she left the glasses. She looks for them, but each time she looks in a place the glasses are actually located, she misses finding them with probability 0.1, whether or not she already looked in the same place. (After all, she doesn't have her glasses on and she is in a hurry.)
(a) Given that Professor Plum didn't find the glasses in her drawer after looking one time, what is the conditional probability the glasses are on the table?
(b) Given that she didn't find the glasses after looking for them in the drawer and on the table once each, what is the conditional probability they are in the briefcase?
(c) Given that she failed to find the glasses after looking in the drawer twice, on the table twice, and in the briefcase once, what is the conditional probability she left the glasses at the office?

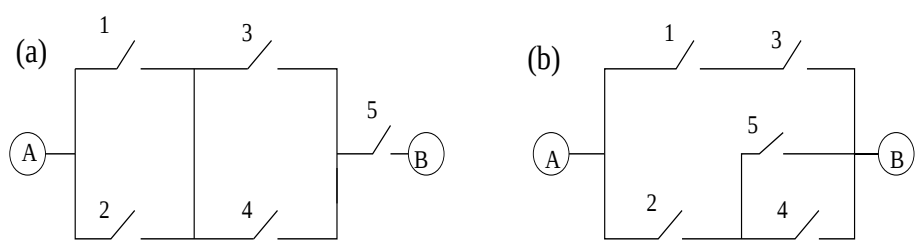
5. Let's make a deal

Dilbert is a contestant on Monty Hall's "Let's Make a Deal" and in the final round, Monty shows him 3 curtains. One of the curtains conceals a prize, the other two, junk. Dilbert selects a curtain at random. (a) At this point in the game, as always, Monty, who knows where the prize is, opens one of the remaining curtains to reveal junk. (If both the other curtains cover junk, he selects one at random with equal probability.) He then offers Dilbert the "new improved deal" — Dilbert can either stick to his original guess or he can switch. For simplicity, we consider that Dilbert made up his mind about what strategy to use before he started to play. What is Dilbert's (absolute) probability of winning the prize (i) if he uses the "stick to original guess" strategy? (ii) if he uses the "switch" strategy?

(b) In a different version this game, Monty calls Wally down from the audience to join Dilbert and asks each of them to pick a curtain. (They must pick different curtains, otherwise all choices are equally likely.) Monty opens one of the two curtains *that was picked*, revealing junk behind it and sends that person back to the audience. (If both contestants picked a curtain with junk, Monty selects one of the two at random, each with equal probability). The other player is now offered the choice of staying with his original choice or switching to the remaining curtain. Compute the probability that Dilbert wins if before he starts play he decides he will (i) use the "stick to original guess" strategy? (ii) use the "switch" strategy? *Note:* Assume for part (b) that Monty always opens one of the two curtains picked by his contestants. In some cases, one of the two curtains conceals the prize and so he is forced to open the other. If neither curtain conceals the prize then he opens one at random.

6. Circuit relay

The state of each switch in the circuits shown is open or closed. Assume that the states of the switches are independent, and that the ith switch is closed with probability p_i for $1 \leq i \leq 5$. Express the probability that there is a path from A to B in terms of p_1, p_2, p_3, p_4, and p_5 for each circuit. (A path can cross a closed switch but not an open switch).



7. Random selection of a card

An ordinary deck of 52 cards is shuffled and then cards are overturned one at a time until the first time an ace appears, and then one more card is overturned. (a) (i) Given that the first ace is the 4th card to appear, what is the conditional probability that the card following it is the Two of clubs? (ii) Given that the first ace is the 30th card to appear, what is the conditional probability that the card following it is the Two of clubs? (iii) What is the (unconditional) probability that the card following the first ace is the Two of Clubs. (Hints: Try taking S=the set of all 52! possible orderings of the cards. Try solving (iii) directly, without using results of (i) and (ii). The answers have a fairly simple form.)

(b) Same as part (a), but replace "Two of Clubs" by "Ace of Spades."