University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 313
Hour Exam # 2

Monday, November 11, 2002
7:00 p.m. — 8:00 p.m.
Room 269 Everitt Laboratory

Name ______________________________

Section:

☐ C 10 MWF
☐ D 11 MWF

INSTRUCTIONS

This exam is closed book and closed notes, except that one 8.5″ × 11″ sheet of notes (both sides) is allowed. Calculators, laptop computers, tables of integrals, etc., are not permitted. The exam consists of 4 problems, with each part of each subproblem worth 5 points for a total of 70 points. Write your answers in the spaces provided. Show *all* your work; if you need extra space, please use the back of the previous page. Partial credit will be given only for substantial progress on a problem. Be careful!

Grading

1.	15 points	______
2.	20 points	______
3.	20 points	______
4.	15 points	______
	TOTAL	______

1. Let X be a random variable with pdf

$$f(x) = \begin{cases} \frac{1}{ax} & \text{if } 1 \leq x \leq 4 \\ 0 & else. \end{cases}$$

(a) Find the constant a and carefully sketch the given pdf $f(x)$.

(b) Find $E[X]$.

(c) Find $P[3 \leq X \leq 5]$.

2. Let U be a continuous random variable uniformly distributed on the interval $[-1, 1]$. Let $X = U^2$. (Hint: don't necessarily solve the subparts in the order given.)

(a) Find the probability density function (pdf) of X, and sketch it. Be sure to determine the function everywhere it is defined.

(b) Find the cumulative distribution function (CDF) of X, and sketch it. Be sure to determine the function everywhere it is defined.

(c) Find $E[X]$.

(d) Find $Var(X)$.

3. One of two hypotheses is to be chosen based on an observation X. If H_0 is the true hypothesis then X is exponentially distributed with parameter $\mu = 5$ (So if H_0 is true the expected value of X is 0.2.) If H_1 is true then X is uniformly distributed on the interval $[0, 1]$.
(a) Find the likelihood ratio $\Lambda(x)$ for $0 \leq x < \infty$. (Hint: What is the value for $x > 1$?)

(b) For what values of X with $0 < X < \infty$ does the maximum likelihood decision rule declare that H_1 is true?

(c) Find the conditional probability of a false alarm given that H_0 is true, p_{false_alarm}.

(d) Find the conditional probability of a miss given that H_1 is true, p_{miss}.

4. Midland Elementary School has an enrollment of 400 children. Assume that each child is equally likely to be a boy or a girl. Ignore leap years and assume that the birthdays of the children are independent and each birthday takes on the 365 possible values with equal probability. You are to give approximate numerical answers to the questions of this problem, and show your work. The following tables for $Q(x) = 1 - \Phi(x)$ and $exp(-x)$ may be useful.

x	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4
$Q(x)$	.3085	.2743	.2420	.2119	.1841	.1587	.1357	.1151	.0968	.0808

x	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4
$exp(-x)$	.6065	.5488	.4966	.4493	.4066	.3679	.3329	.3012	.2725	.2466

(a) What is the approximate probability that at least 205 of the children are girls?

(b) What is the approximate probability that exactly two of the children have birthday May 15?

(c) What is the approximate probability that none of the *girls* has birthday June 1?