**Assigned:** Wednesday, November 7, 2001
**Due:** Wednesday, November 14, 2001

**Reminder:** Hour Exam II is scheduled for Monday November 12 from 11:00 a.m. to 11:50 a.m. in Room 260 EL, that is, in the usual classroom at the usual meeting tome. Consult the web page for details of coverage.

**Reading:** Ross, Chapter 6 (except Sections 6.6 and 6.8)
**Noncredit Exercises:** Ross, Chapter 6: Problems 1, 10-15, 20-23
**Problems:**

**1.** The discrete random variables **X** and **Y** have joint pmf $p_{X,Y}(u,v)$ given by

| | | | | |
|---|---|---|---|---|
| 4 | 0 | 1/12 | 1/6 | 1/12 |
| 3 | 1/6 | 1/12 | 0 | 1/12 |
| –1 | 1/12 | 1/6 | 1/12 | 0 |
| v↑ / u→ | 0 | 1 | 3 | 5 |

**(a)** Find the marginal pmfs $p_X(u)$ and $p_Y(v)$ of **X** and **Y**.
**(b)** Are the random variables **X** and **Y** independent ?
**(c)** Find $P\{X \le Y\}$ and $P\{X + Y \le 8\}$.
**(d)** Find $p_{X|Y}(u|3)$, $E[X|Y=3]$, and $\mathrm{var}(X|Y=3)$.

**2.** Ross, Chapter 6, Problem 8

**3.** Ross, Chapter 6, Problem 9

**4.** We return to the “random chord” of Problem 1 of Problem Set #11. Yet another way of defining a “random chord” is to choose the midpoint of the chord to be anywhere inside the circle with equal probability. The chord is, of course, perpendicular to the diameter that passes through the point. Thus, let the random point $(X, Y)$ be uniformly distributed on the interior of the circle of unit radius centered at the origin (this region is called the unit disc — nomenclature that might be familiar to DSPists).
**(a)** Find the probability that the length **L** of the random chord is greater than the side of the equilateral triangle inscribed in the circle.
**(b)** Express **L** as a function of the random variable $(X, Y)$ and find the probability density function for **L**.
**(c)** Find the average length of the chord, i.e. find $E[L]$.

**5.** The jointly continuous random variables **X** and **Y** have joint pdf given by

$$f_{X,Y}(u,v) = \begin{cases} 2\exp -(u+v), & 0 < u < v < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

**(a)** Sketch the u-v plane and indicate on it the region over which $f_{X,Y}(u,v)$ is nonzero.
**(b)** Find the marginal pdfs of **X** and **Y**.
**(c)** Are the random variables **X** and **Y** independent ?
**(d)** Find $P\{Y > 3X\}$.
**(e)** For $\alpha > 0$, find $P\{X + Y \le \alpha\}$.
**(f)** Use the result in part (e) to determine the pdf of the random variable $Z = X + Y$.

**6.** The jointly continuous random variables **X** and **Y** have joint pdf

$$f_{X,Y}(u,v) = \begin{cases} 1/2, & 0 \le u < 1,\ 0 \le v < 1, \text{ and } 0 \le u + v < 1 \\ 3/2, & 0 \le u < 1,\ 0 \le v < 1, \text{ and } 1 \le u + v < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find $f_X(u)$, $P\{X + Y \le 3/2\}$ and $P\{X^2 + Y^2 \ge 1\}$.