University of Illinois | **Problem Set #3** | **ECE 313**

 | **Fall 2001**

Assigned: Wednesday, September 5
Due: Wednesday, September 12
Reading: Ross, Chapters 2.1–2.5, 4.1, 4.3–4.5, 4 .7
Noncredit Exercises: (Do not turn these in) Chapter 4, Problems: 2, 7, 13, 28, 35, 39, 40-43; Theoretical Exercises: 11, 13, 15.

Problems:

1.(a) Events A and B are **disjoint** events defined on a sample space. What is the probability that at least one of the two events A and B did not occur? If the answer cannot be determined from the given information, say so explicitly.

(b) Events C and D are events defined on a sample space. If the probability that at least one of the two events occurred is 0.6, and the probability that at least one of the events did not occur is 0.8, what is the probability that *exactly one* of the events C and D did not occur? If the answer cannot be determined from the given information, say so explicitly.

2. A certain town has three newspapers A, B, and C. The proportions of townspeople that read these newspapers are as follows:
A: 10%, B: 30%, C: 5%; A and B: 8%, A and C: 2 %, B and C: 4%, and 1% read all three newspapers.

(a) What percentage of people read only one newspaper?
(b) What percentage of people read at least two newspapers?
(c) If A and C are morning papers and B is an evening paper, what percentage of people read at least one morning paper as well as the evening paper?
(d) How many people do not read any newspapers?

3. The experiment consists of picking a student from the set of all UIUC students registered this semester. It is **not** necessary to assume that all students are equally likely to be picked, but you may make this assumption if it makes you feel happier and more confident.

(a) Let A and B denote the events that the student picked has had respectively four years of science (FYS) and calculus in high school. Let P(A) = 0.45 and P(B) = 0.35. If the probability that the student had neither FYS nor calculus is 0.3, what is the probability that the student had both FYS **and** calculus? What is the probability that the student had FYS but **not** calculus ?

(b) Let C denote the event that the student is registered in ECE 313, and let A and B be as in part (a). Suppose that $P(A\cap B\cap C)=0.002$. What is the probability that the student picked is not registered in ECE 313, but did have both FYS **and** calculus ? If the probability that the student picked is registered in ECE 313, and has had either FYS or calculus (but not both) is 0.004, and if students who had neither FYS nor calculus did not register in ECE 313, what is P(C)?

(c) Using the data given in parts (a) and (b), which of the following probabilities can you compute? It is not necessary to actually compute each probability.

P(A   C), P(A B C), P(A   B C^c), $P(A^cB^cC^c)$, $P(A^cBC^c)$, $P(ABC^c)$

4. Let A, B, C denote three events defined on a sample space. Show that

$$\frac{P(A) + P(B) + P(C)}{3} \leq P(A \quad B \quad C) \leq P(A) + P(B) + P(C)$$

5. Find P(A   (B^c C^c)^c) in each of the following four cases:
(a) A, B, and C are mutually exclusive events and P(A) = 1/3.
(b) P(A) = 2P(BC) = 4P(ABC) = 1/2.
(c) P(A) = 1/2, P(BC) = 1/3, and P(AC) = 0.
(d) $P(A^c \cap (B^c \quad C^c)) = 0.6$.

6. Borrowing an idea from the Big Ten, the NCAA Basketball Tournament has *five* teams in its semifinal round (Final Four). Each team plays the other four teams exactly once in the semifinal round.

(a) What is the total number of games played in the semifinal round?

(b) In each game, one team wears a dark uniform while the other wears a light uniform. Is it possible to arrange matters such that each team wears dark uniforms for two of its games and light uniforms for the other two?

(c) No game ends in a tie; one team wins and the other loses. If n denotes your answer to part (a), then there are 2^n different results that might occur, and we assume that all of the 2^n results are equally likely. With this assumption, find the probability that each team wins at least one game and loses at least one game (that is, no team has a 4-0 or 0-4 record in the semifinal round.)

7. Use a spreadsheet/Mathematica/MATLAB for this problem.
Let A denote an event of probability p.

(a) For p = 0.1, 0.25. 0.4, 0.5, 0.6, 0.75, and 0.9, find the numerical values of the probabilities that A occurs 0, 1, 2, … , 10 times on 10 trials of the experiment.

(b) You have, in effect, computed the probability mass function for a binomial random variable **X** with parameters (10, p) for seven choices of p. For each value of p, draw a bar graph of the pmf. (For p = 0.5, the answer is on page 151 (5th ed.) or 146 (6th ed.)of Ross!)

(c) What is the relationship between the pmfs for the cases p = 0.1 and p = 0.9? for the cases p = 0.25 and p = 0.75? for the cases p = 0.4 and 0.6?

(d) Prove mathematically that if **X** is a binomial random variable with parameters (n, p), then **Y** = n – **X** is a binomial random variable with parameters (n, 1–p).

(e) From each of the seven graphs of part (b), find the value of k for which P{**X** = k} is maximum. Compare your results to the prediction of Proposition 7.1, p. 150, of Ross.

8. [“Eat your broccoli, dear. It’s good for you”] Let A, B, and C denote the events that your mother serves respectively asparagus, broccoli, and cauliflower for dinner. From (bitter?) experience you know that these events are mutually exclusive (i.e. you get only one vegetable each day) and that P(A) = 0.2, P(B) = 0.5, and P(C) = 0.3. Each day is an independent trial: that is, your mother, a lady of formidable temperament albeit limited culinary skills, makes *independent* decisions (e.g. without taking into account your opinion that Cheetos is a vegetable suitable for serving with any entree) about the vegetable to serve each day. Over a three day period, what is the probability that

(a) she serves the same vegetable on all three days ?

(b) she serves the same vegetable exactly two days out of three ?

(c) she serves different vegetables on the three days ?