Conditional Distributions

Recall:

The probability that \((X, Y)\) lies in a differential region is:

\[
P(x < a \& y < b) = \int_a^b f(x, y) \, dy \int f(x, y) \, dx
\]

That is,

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Suppose we observe \(X = \chi\): how to condition on this

1. For discrete rv's: \(P(X = \chi)\)

2. For continuous rv's: \(f_X(\chi)\) \(f_Y(\chi)\) \(f_X(\chi, y)\)

Finally,

\[
\frac{P(X = \chi \& Y = y)}{P(X = \chi)} = \frac{f_X(\chi, y)}{f_X(\chi)}
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Example

Define \( f_{XY}(u|v) = \frac{f_{X,Y}(u,v)}{f_Y(v)} \)

for \( f_Y(v) \neq 0 \)

Note that \( f_{X|Y}(u|v) = \frac{f_X(U|Y)}{\text{known}} \)

\( f_{X|Y}(u|v) \) is a known variable.

\[ f_{X|Y}(u|v) = \begin{cases} 1 & \text{if } 0 < u < 1 \\ 0 & \text{otherwise} \end{cases} \]

Example

\[ X \sim \mathcal{U}(0,1), \quad Y \sim \mathcal{U}(0,1) \]

\[ f_{X,Y}(x,y) = e^{-(y-x)} \quad \forall x,y \in [0,1] \]

\[ f_{X|Y}(x|y) = e^{-y} \quad \text{for } 0 < x < y < 1 \]

\[ f_{Y|X}(y|x) = \frac{1 - y}{1 - x} \quad \text{for } x < y < 1 \]

\[ M(\rho, 1 - \rho^2) \]