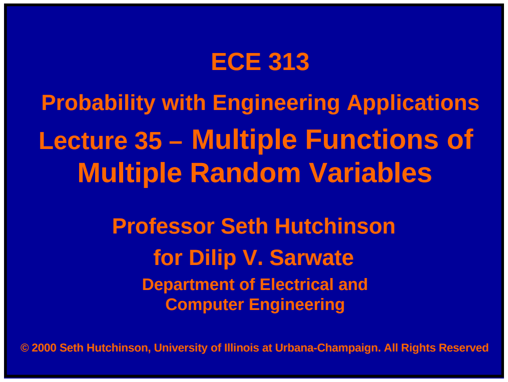
ECE 313
Probability with Engineering Applications
Lecture 35 – Multiple Functions of Multiple Random Variables
Professor Seth Hutchinson
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Department of Electrical and Computer Engineering
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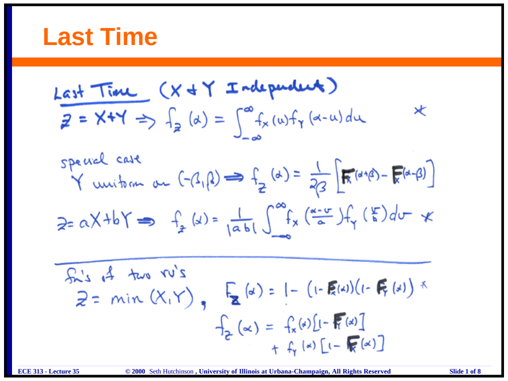
Last Time
Last Time (X + Y Independent)
special case
fn's of two rv's
Z = min (X, Y)
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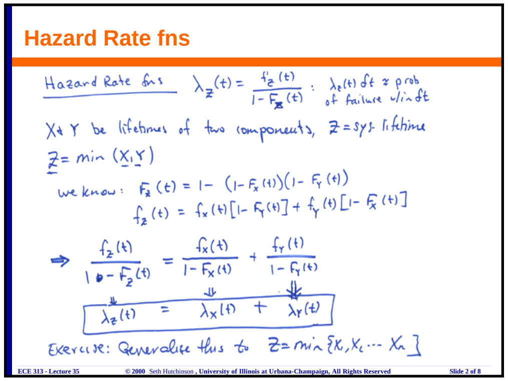
Hazard Rate fns
Hazard Rate fns
X, Y be lifetimes of two components, Z = sys lifetime
Z = min (X, Y)
we know:
Exercise: Generalize this to Z = min {X1, X2 ... Xn}
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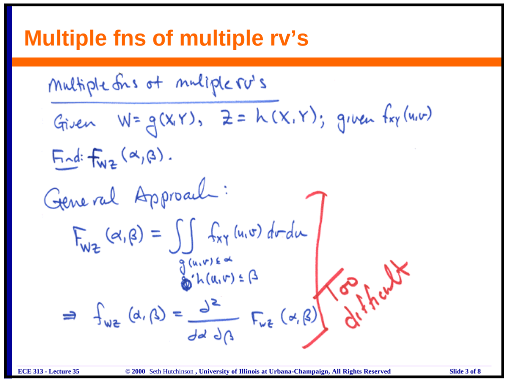
Multiple fns of multiple rv's
Multiple fns of multiple rv's
Given W = g(X, Y), Z = h(X, Y); given fXY(u, v)
Find: fWZ(α, β).
General Approach:
Too difficult
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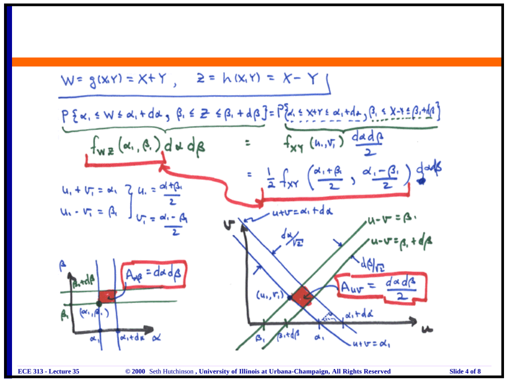
W = g(X,Y) = X+Y, Z = h(X,Y) = X − Y
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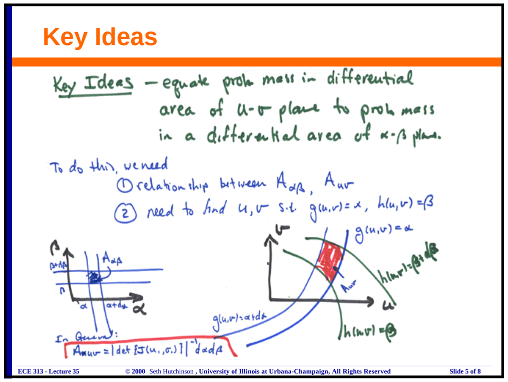
Key Ideas
Key Ideas – equate prob mass in differential area of u-v plane to prob mass in a differential area of α-β plane.
To do this, we need
① relationship between Aαβ, Auv
② need to find u, v s.t. g(u,v) = α, h(u,v) = β
In General:
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$$A_{uv} = \left| \det \{J(u_i, v_i)\} \right|^{-1} \underbrace{d\alpha\, d\beta}_{A_{\alpha\beta}}$$

where: $g(u_i, v_i) = \alpha$, $h(u_i, v_i) = \beta$

$$J(u,v) = \begin{bmatrix} \partial g/\partial u & \partial g/\partial v \\ \partial h/\partial u & \partial h/\partial v \end{bmatrix}$$

$$\Rightarrow f_{WZ}(\alpha, \beta) = \sum_i \frac{f_{XY}(u_i, v_i)}{|\det J(u_i, v_i)|}$$

for i ranging over all solns $g(u_i\, v_i) = \alpha$, $h(u_i\, v_i) = \beta$.

$g(X, Y) = X + Y$, $h(X, Y) = X - Y$

$$J = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow \left| \det \{J\} \right| = |-2| = 2$$

$$\underline{u} = \frac{\alpha+\beta}{2}, \quad \underline{v} = \frac{\alpha-\beta}{2}$$

$$f_{WZ}(\alpha, \beta) = \frac{1}{2} f_{XY}\left(\frac{\alpha+\beta}{2}, \frac{\alpha-\beta}{2}\right)$$

<u>It works when</u>

- we can solve g, h for u_i, v_i
- $g + h$ have cont. partial ~~derivatives~~ der's
  $\det \{J\} \neq 0$