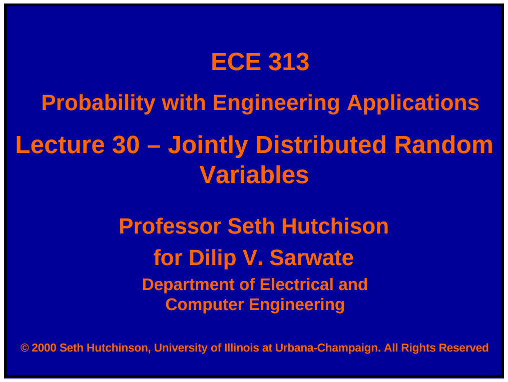
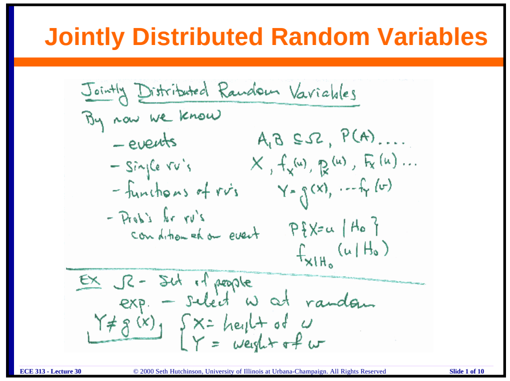
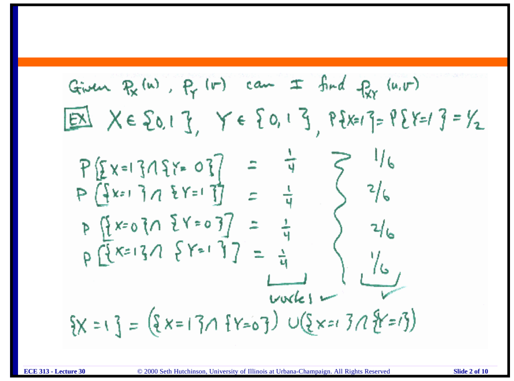
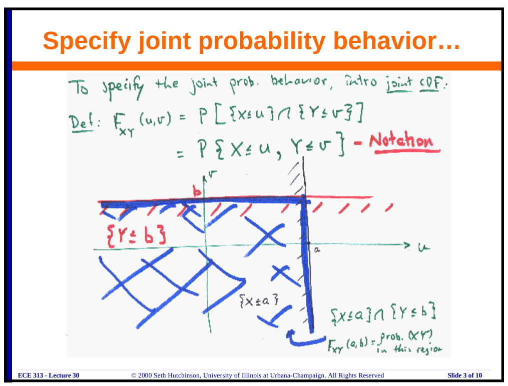
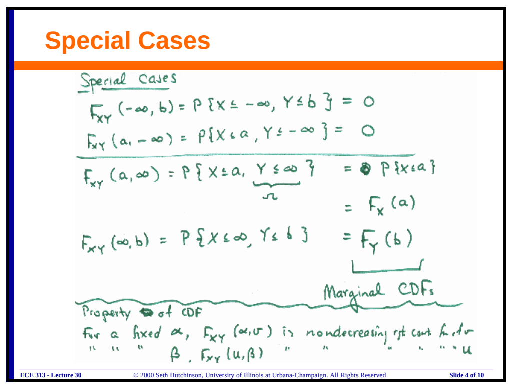
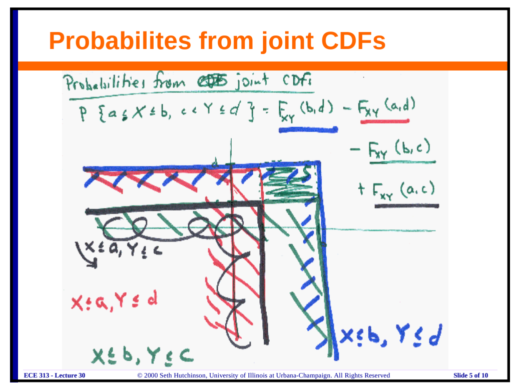

ECE 313
Probability with Engineering Applications
Lecture 30 – Jointly Distributed Random Variables
Professor Seth Hutchison
for Dilip V. Sarwate
Department of Electrical and Computer Engineering
© 2000 Seth Hutchinson, University of Illinois at Urbana-Champaign. All Rights Reserved

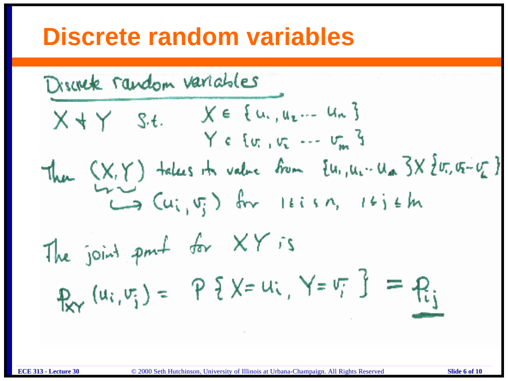

Jointly Distributed Random Variables

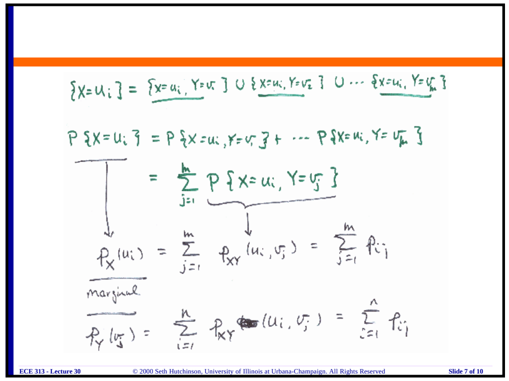


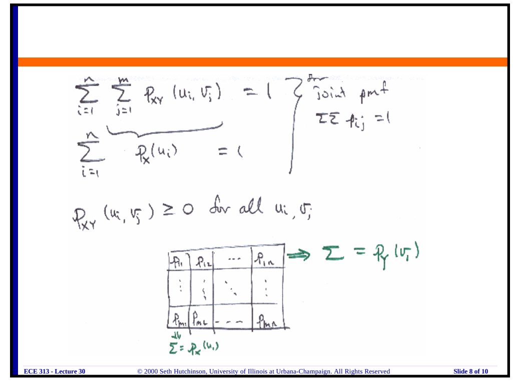

Specify joint probability behavior…

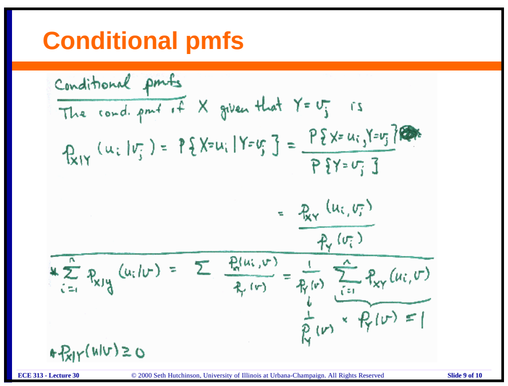

Special Cases

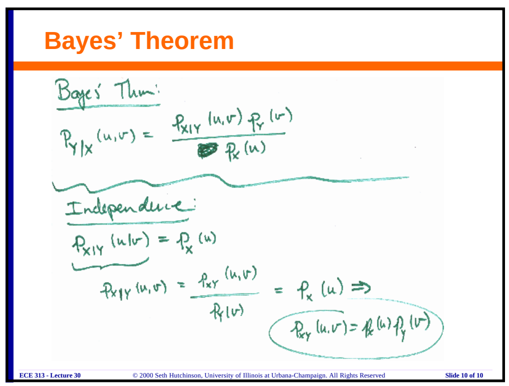

Probabilites from joint CDFs


Discrete random variables
Discrete random variables
$X + Y$ s.t. $X \in \{u_1, u_2 \cdots u_n\}$
$Y \in \{v_1, v_2 \cdots v_m\}$
Then (X, Y) takes its value from $\{u_1, u_2 \cdots u_n\} \times \{v_1, v_2 \cdots v_m\}$
$\hookrightarrow (u_i, v_j)$ for $1 \le i \le n$, $1 \le j \le m$
The joint pmf for XY is
$p_{XY}(u_i, v_j) = P\{X = u_i, Y = v_j\} = p_{ij}$
ECE 313 - Lecture 30
© 2000 Seth Hutchinson, University of Illinois at Urbana-Champaign. All Rights Reserved
Slide 6 of 10


$\{X = u_i\} = \{X = u_i, Y = v_1\} \cup \{X = u_i, Y = v_2\} \cup \cdots \{X = u_i, Y = v_m\}$
$P\{X = u_i\} = P\{X = u_i, Y = v_1\} + \cdots P\{X = u_i, Y = v_m\}$
$= \sum_{j=1}^{m} P\{X = u_i, Y = v_j\}$
$p_X(u_i) = \sum_{j=1}^{m} p_{XY}(u_i, v_j) = \sum_{j=1}^{m} p_{ij}$
marginal
$p_Y(v_j) = \sum_{i=1}^{n} p_{XY}(u_i, v_j) = \sum_{i=1}^{n} p_{ij}$
ECE 313 - Lecture 30
© 2000 Seth Hutchinson, University of Illinois at Urbana-Champaign. All Rights Reserved
Slide 7 of 10


$\sum_{i=1}^{n} \sum_{j=1}^{m} p_{XY}(u_i, v_j) = 1$
$\sum_{i=1}^{n} p_X(u_i) = 1$
for joint pmf
$\Sigma\Sigma\, p_{ij} = 1$
$p_{XY}(u_i, v_j) \ge 0$ for all u_i, v_j
p_{11} p_{12} $\cdots$ p_{1n}
p_{m1} p_{m2} $\cdots$ p_{mn}
$\Rightarrow \Sigma = p_Y(v_1)$
$\Sigma = p_X(u_1)$
ECE 313 - Lecture 30
© 2000 Seth Hutchinson, University of Illinois at Urbana-Champaign. All Rights Reserved
Slide 8 of 10


Conditional pmfs
Conditional pmfs
The cond. pmf of X given that $Y = v_j$ is
$p_{X|Y}(u_i|v_j) = P\{X = u_i | Y = v_j\} = \frac{P\{X = u_i, Y = v_j\}}{P\{Y = v_j\}}$
$= \frac{p_{XY}(u_i, v_j)}{p_Y(v_j)}$
$* \sum_{i=1}^{n} p_{X|Y}(u_i|v) = \Sigma \frac{p_{XY}(u_i, v)}{p_Y(v)} = \frac{1}{p_Y(v)} \sum_{i=1}^{n} p_{XY}(u_i, v)$
$\frac{1}{p_Y(v)} \times p_Y(v) = 1$
$* p_{X|Y}(u|v) \ge 0$
ECE 313 - Lecture 30
© 2000 Seth Hutchinson, University of Illinois at Urbana-Champaign. All Rights Reserved
Slide 9 of 10


Bayes' Theorem
Bayes' Thm:
$p_{Y|X}(u, v) = \frac{p_{X|Y}(u, v)\, p_Y(v)}{p_X(u)}$
Independence:
$p_{X|Y}(u|v) = p_X(u)$
$p_{X|Y}(u, v) = \frac{p_{XY}(u, v)}{p_Y(v)} = p_X(u) \Rightarrow$
$p_{XY}(u, v) = p_X(u)\, p_Y(v)$
ECE 313 - Lecture 30
© 2000 Seth Hutchinson, University of Illinois at Urbana-Champaign. All Rights Reserved
Slide 10 of 10