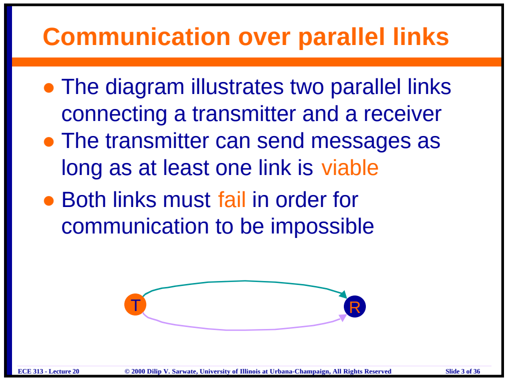
Communication Networks

- Communication networks consist of links over which messages can pass between nodes (or terminals or hosts)
- Messages to distant nodes have to pass over multiple links and multiple nodes
- The intermediate links can fail
- The intermediate nodes can fail
- What is the probability that two nodes can communicate?

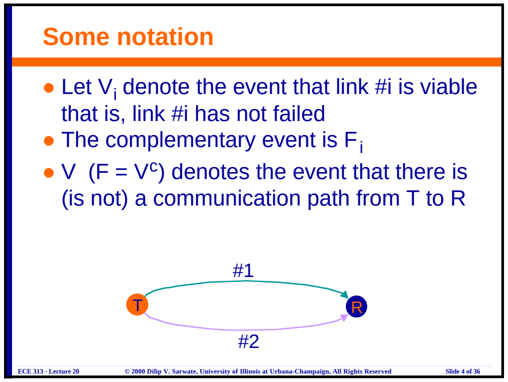
Links only; we don’t do nodes

- A node that fails makes all the links connected to it inoperable
- All paths between other nodes that pass through the failed node are not available
- In the simple analyses presented in this course, we assume that nodes do not fail
- The only failures that we are concerned with are the link failures



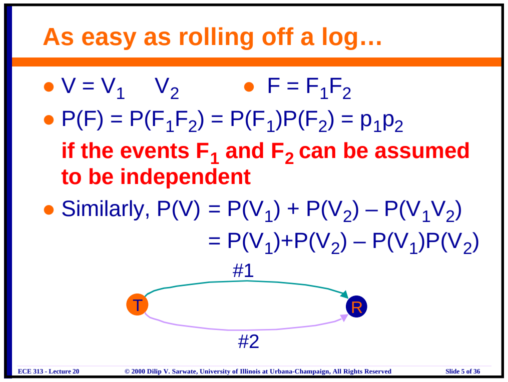
Communication over parallel links

- The diagram illustrates two parallel links connecting a transmitter and a receiver
- The transmitter can send messages as long as at least one link is viable
- Both links must fail in order for communication to be impossible



Some notation

- Let V_i denote the event that link #i is viable that is, link #i has not failed
- The complementary event is F_i
- V (F = V^c) denotes the event that there is (is not) a communication path from T to R



As easy as rolling off a log…

- $V = V_1 \quad V_2$
- $F = F_1F_2$
- $P(F) = P(F_1F_2) = P(F_1)P(F_2) = p_1p_2$

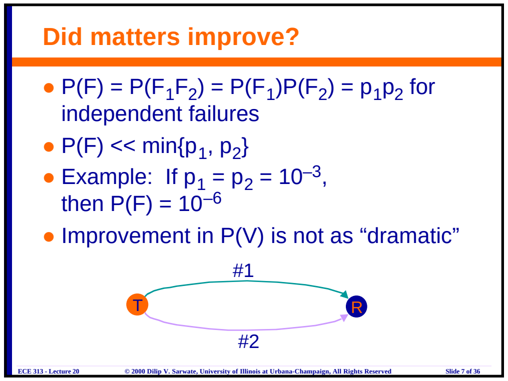
  if the events F_1 and F_2 can be assumed to be independent
- Similarly, $P(V) = P(V_1) + P(V_2) – P(V_1V_2)$

  $= P(V_1)+P(V_2) – P(V_1)P(V_2)$

Let’s be sure we understand…

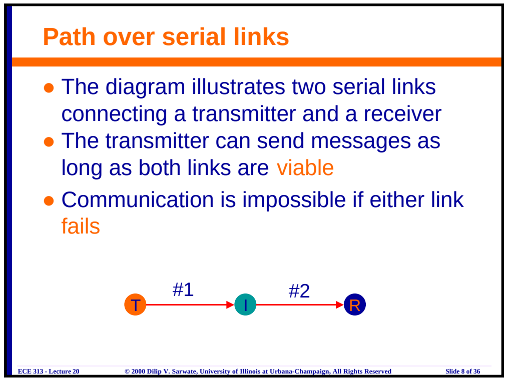
- $V = V_1 \quad V_2$
- $F = F_1F_2$
- $P(F) = P(F_1F_2) = P(F_1)P(F_2) = p_1p_2$

  if the events F_1 and F_2 can be assumed to be independent
- $$\begin{aligned} P(V) &= P(V_1) + P(V_2) – P(V_1V_2) \\ &= P(V_1) + P(V_2) – P(V_1)P(V_2) \\ &= q_1 + q_2 – q_1q_2 \\ &= (1 – p_1) + (1 – p_2) – (1 – p_1)(1 – p_2) \\ &= 1 – p_1p_2 \end{aligned}$$



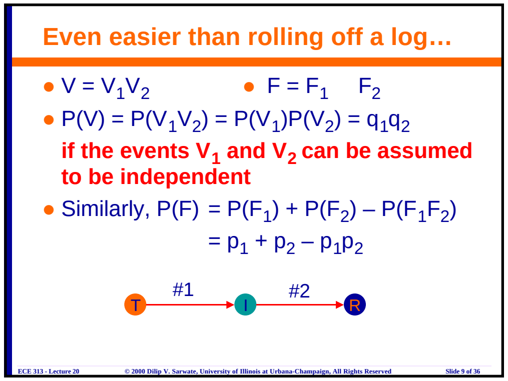
Did matters improve?

- $P(F) = P(F_1F_2) = P(F_1)P(F_2) = p_1p_2$ for independent failures
- $P(F) << \min\{p_1, p_2\}$
- Example: If $p_1 = p_2 = 10^{-3}$, then $P(F) = 10^{-6}$
- Improvement in P(V) is not as “dramatic”



Path over serial links

- The diagram illustrates two serial links connecting a transmitter and a receiver
- The transmitter can send messages as long as both links are viable
- Communication is impossible if either link fails

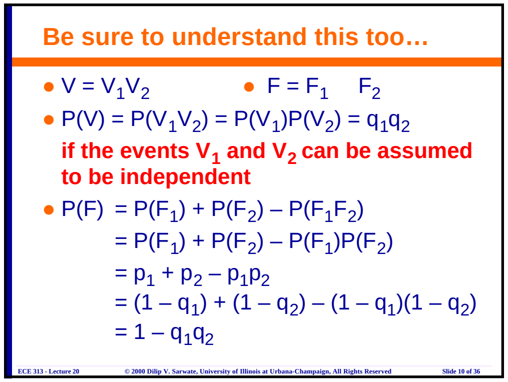


Even easier than rolling off a log…

- $V = V_1V_2$
- $F = F_1 \quad F_2$
- $P(V) = P(V_1V_2) = P(V_1)P(V_2) = q_1q_2$

if the events V_1 and V_2 can be assumed to be independent

- Similarly, $P(F) = P(F_1) + P(F_2) – P(F_1F_2)$
  $= p_1 + p_2 – p_1p_2$



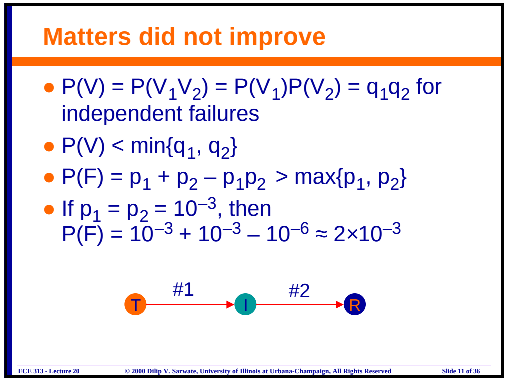
Be sure to understand this too…

- $V = V_1V_2$
- $F = F_1 \quad F_2$
- $P(V) = P(V_1V_2) = P(V_1)P(V_2) = q_1q_2$

if the events V_1 and V_2 can be assumed to be independent

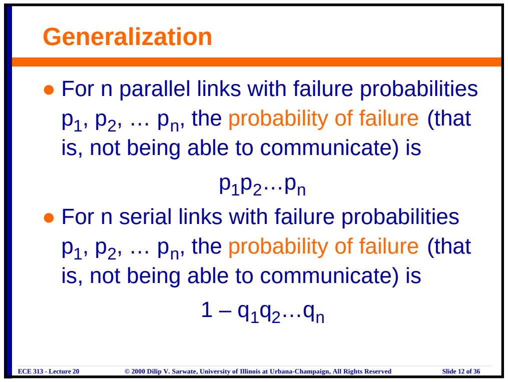
- $P(F) = P(F_1) + P(F_2) – P(F_1F_2)$

$$\begin{aligned} &= P(F_1) + P(F_2) - P(F_1)P(F_2) \\ &= p_1 + p_2 - p_1p_2 \\ &= (1 - q_1) + (1 - q_2) - (1 - q_1)(1 - q_2) \\ &= 1 - q_1q_2 \end{aligned}$$



Matters did not improve

- $P(V) = P(V_1V_2) = P(V_1)P(V_2) = q_1q_2$ for independent failures
- $P(V) < \min\{q_1, q_2\}$
- $P(F) = p_1 + p_2 – p_1p_2 > \max\{p_1, p_2\}$
- If $p_1 = p_2 = 10^{-3}$, then $P(F) = 10^{-3} + 10^{-3} - 10^{-6} \approx 2\times10^{-3}$



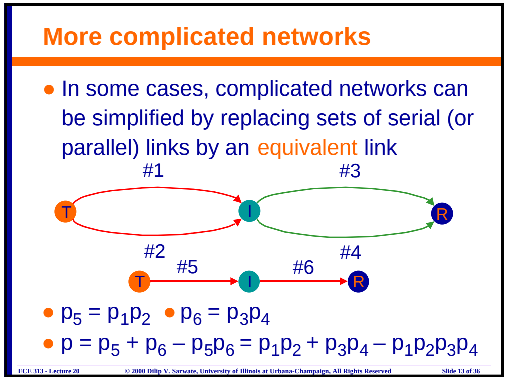
Generalization

- For n parallel links with failure probabilities $p_1, p_2, \ldots p_n$, the probability of failure (that is, not being able to communicate) is

$$p_1p_2\ldots p_n$$

- For n serial links with failure probabilities $p_1, p_2, \ldots p_n$, the probability of failure (that is, not being able to communicate) is

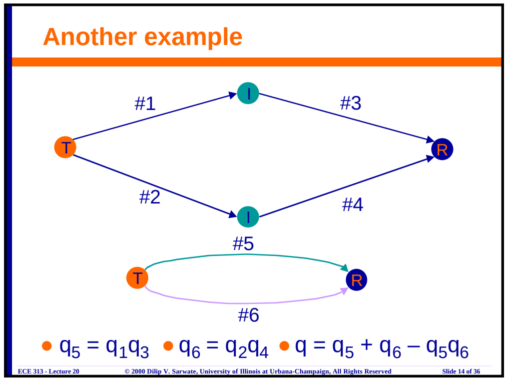
$$1 - q_1q_2\ldots q_n$$



More complicated networks

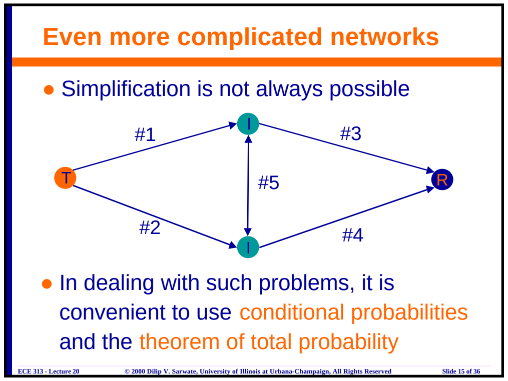
- In some cases, complicated networks can be simplified by replacing sets of serial (or parallel) links by an equivalent link

- $p_5 = p_1p_2$
- $p_6 = p_3p_4$
- $p = p_5 + p_6 - p_5p_6 = p_1p_2 + p_3p_4 - p_1p_2p_3p_4$



Another example

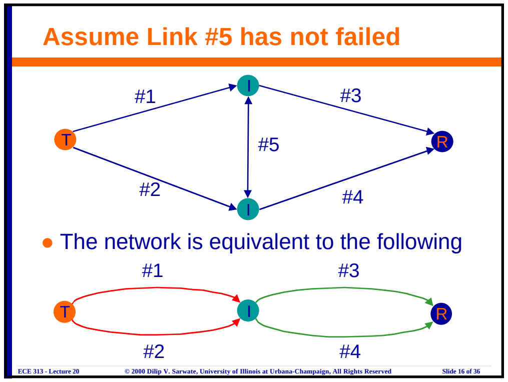
- $q_5 = q_1q_3$
- $q_6 = q_2q_4$
- $q = q_5 + q_6 - q_5q_6$



Even more complicated networks

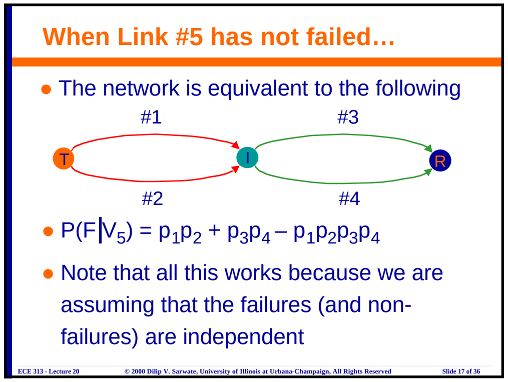
- Simplification is not always possible

- In dealing with such problems, it is convenient to use conditional probabilities and the theorem of total probability



Assume Link #5 has not failed

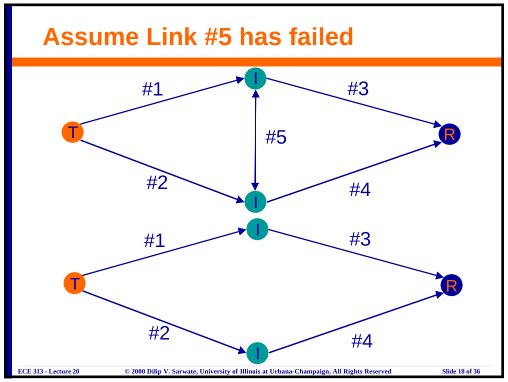
- The network is equivalent to the following



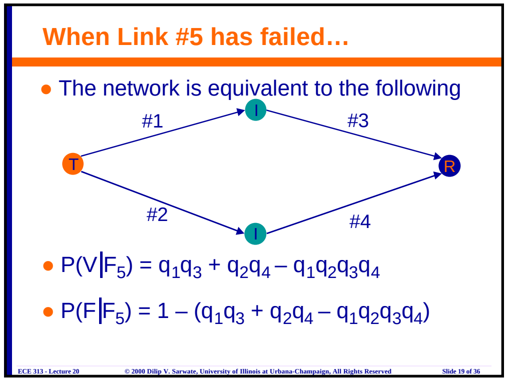
When Link #5 has not failed…

- The network is equivalent to the following

- $P(F|V_5) = p_1p_2 + p_3p_4 - p_1p_2p_3p_4$
- Note that all this works because we are assuming that the failures (and non-failures) are independent



Assume Link #5 has failed



When Link #5 has failed…

- The network is equivalent to the following

- $P(V|F_5) = q_1q_3 + q_2q_4 - q_1q_2q_3q_4$
- $P(F|F_5) = 1 - (q_1q_3 + q_2q_4 - q_1q_2q_3q_4)$

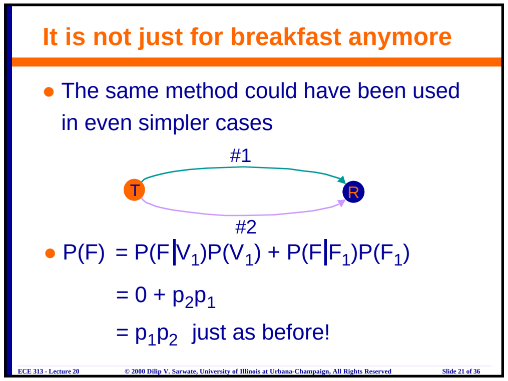
Now put it all together …

- $P(F) = P(F|V_5)P(V_5) + P(F|F_5)P(F_5)$

$$= (p_1p_2 + p_3p_4 - p_1p_2p_3p_4)(1 - p_5)$$

$$+ (1 - (q_1q_3 + q_2q_4 - q_1q_2q_3q_4))p_5$$

- Thus, we see that using conditional probabilities of failure together with the theorem of total probability is a useful trick



It is not just for breakfast anymore

- The same method could have been used in even simpler cases

- $P(F) = P(F|V_1)P(V_1) + P(F|F_1)P(F_1)$

$$= 0 + p_2p_1$$

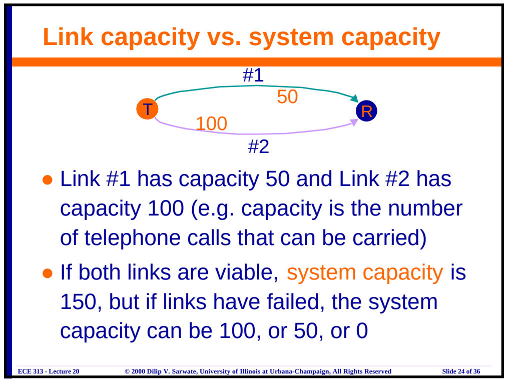
$$= p_1p_2 \text{ just as before!}$$

What is communication capacity?

- An important question in communication networks is how much traffic (e.g. calls) can be carried from one node to another
- The amount of traffic that can be carried is called the communication capacity
- Distinguish between the traffic that can be carried, and the traffic that is being carried
- The latter usually depends on conditions outside the network

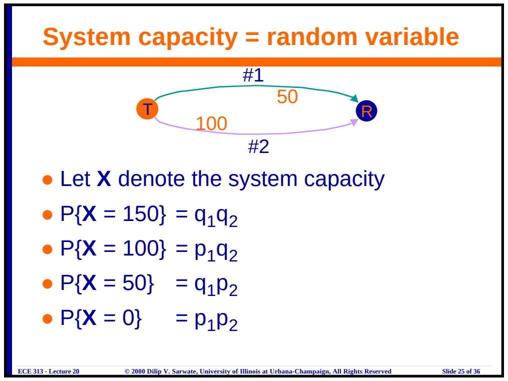
More detailed answers, please!

- Failure analysis reveals only that “Yes, communication is possible” or “No, communication is not possible”
- Failure analysis crudely quantifies the answer to the question of communication capacity into two values: zero and nonzero
- We seek a more detailed description
- Because of link failures, the capacity can be considered to be a random variable



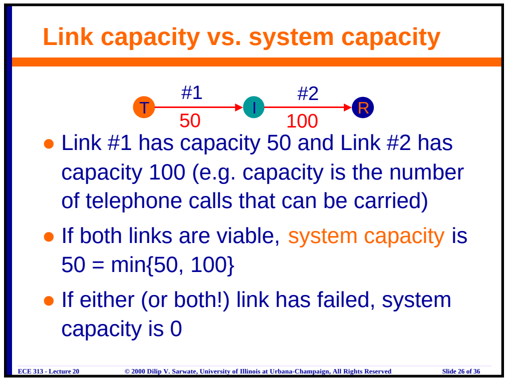
Link capacity vs. system capacity

- Link #1 has capacity 50 and Link #2 has capacity 100 (e.g. capacity is the number of telephone calls that can be carried)
- If both links are viable, system capacity is 150, but if links have failed, the system capacity can be 100, or 50, or 0



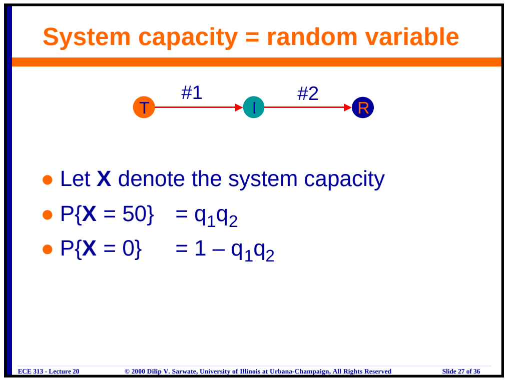
System capacity = random variable

- Let **X** denote the system capacity
- P{**X** = 150} = q_1q_2
- P{**X** = 100} = p_1q_2
- P{**X** = 50} = q_1p_2
- P{**X** = 0} = p_1p_2



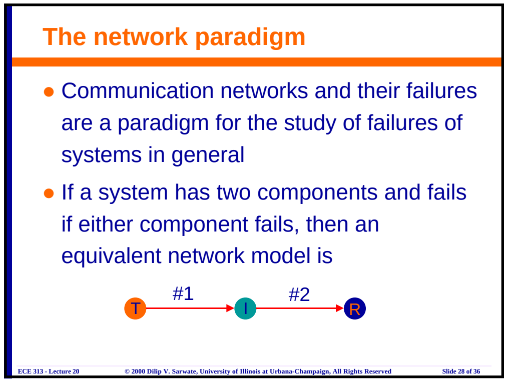
Link capacity vs. system capacity

- Link #1 has capacity 50 and Link #2 has capacity 100 (e.g. capacity is the number of telephone calls that can be carried)
- If both links are viable, system capacity is 50 = min{50, 100}
- If either (or both!) link has failed, system capacity is 0



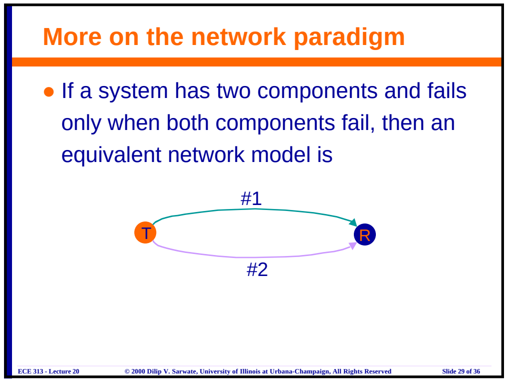
System capacity = random variable

- Let **X** denote the system capacity
- P{**X** = 50} = q_1q_2
- P{**X** = 0} = $1 - q_1q_2$



The network paradigm

- Communication networks and their failures are a paradigm for the study of failures of systems in general
- If a system has two components and fails if either component fails, then an equivalent network model is

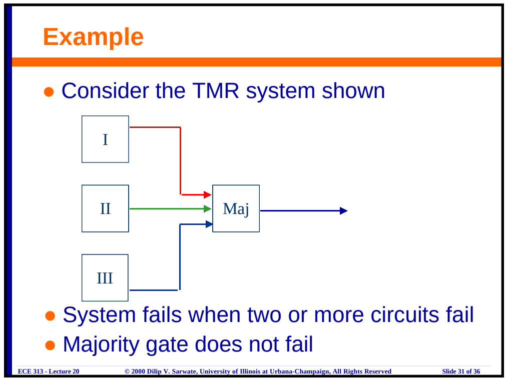


More on the network paradigm

- If a system has two components and fails only when both components fail, then an equivalent network model is

But, complications arise…

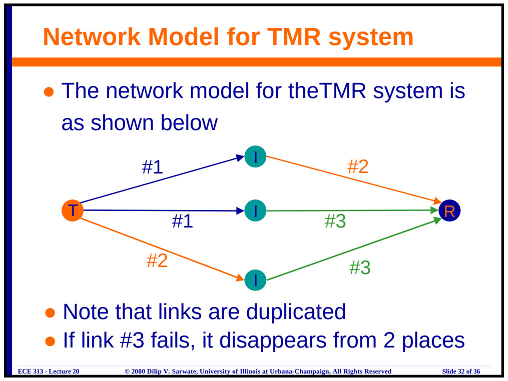
- In modeling some systems as a communication network, it is sometimes necessary to have a link appear in more than one place
- When the corresponding component fails, all these links fail



Example

- Consider the TMR system shown

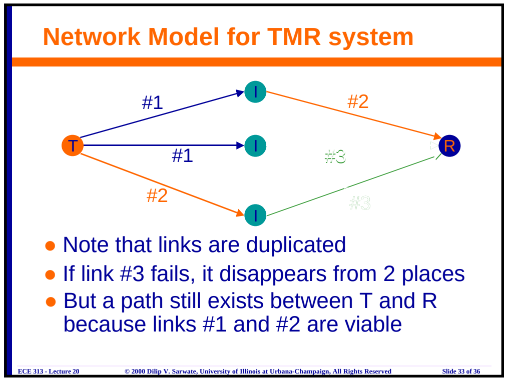
- System fails when two or more circuits fail
- Majority gate does not fail



Network Model for TMR system

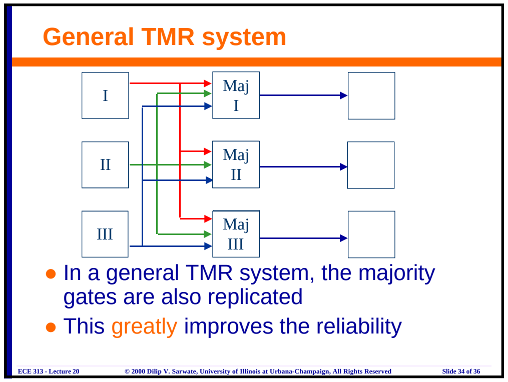
- The network model for theTMR system is as shown below

- Note that links are duplicated
- If link #3 fails, it disappears from 2 places



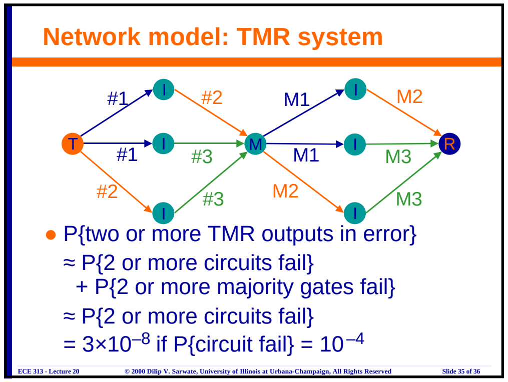
Network Model for TMR system

- Note that links are duplicated
- If link #3 fails, it disappears from 2 places
- But a path still exists between T and R because links #1 and #2 are viable



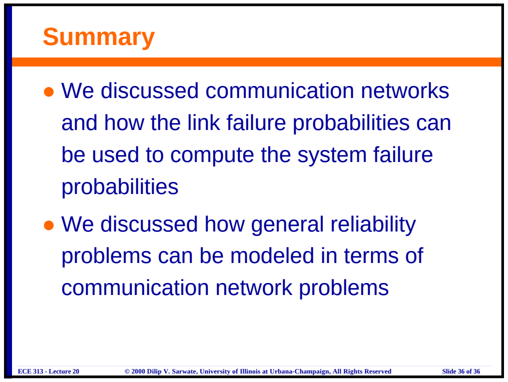
General TMR system

- In a general TMR system, the majority gates are also replicated
- This greatly improves the reliability



Network model: TMR system

- P{two or more TMR outputs in error}
  ≈ P{2 or more circuits fail}
  + P{2 or more majority gates fail}
  ≈ P{2 or more circuits fail}
  = 3×10^{-8} if P{circuit fail} = 10^{-4}



Summary

- We discussed communication networks and how the link failure probabilities can be used to compute the system failure probabilities
- We discussed how general reliability problems can be modeled in terms of communication network problems