Welcome to ECE 313

- ECE 313 is a course on probability theory with
  - Some statistical applications
  - Some applications to selected problems in electrical and computer engineering
- ECE 313 is (semi)-required in the BSEE and BSCompE curricula

What does semi-required mean?

- All BSEE and BSCompE students must take one of the following three courses
  - ECE 313
  - Industrial Engineering 230
  - Statistics 310

How is ECE 313 different?

- ECE 313 is a course on
  - Mathematical probability theory
  - Applications to the statistical problems of
    - decision-making under uncertainty
    - estimation of parameters
  - Applications to problems in engineering
- IE 230 and Stat 310 primarily teach statistical methods

How does ECE 313 compare?

- The coverage of probability is more complete and in more depth in ECE 313 than in IE 230 or Stat 310
- The coverage of statistics in ECE 313 is idiosyncratic and incomplete compared to the broad coverage in IE 230 or Stat 310
- ECE 313 primarily covers some statistical techniques applicable to communications and radar systems

Why take ECE 313 and not others?

- ECE 313 is essential preparation for many senior-level and graduate courses in
  - communication systems
  - signal processing
  - computer networks
  - computer system analysis
  - reliability of systems
- IE 230 and Stat 310 will not prepare you as well for these courses

What can you do after ECE 313?

- Links to courses that have ECE 313 as a prerequisite can be found on the class web page at **http://www2.ece.uiuc.edu/~ece313/faq.html**
- You will be able to apply probabilistic reasoning to understand how and why various statistical procedures work

What is Probability?

- Since pre-historic times, Mankind has been aware of
  - Deterministic phenomena
    - daily sunrises and sunsets
    - tides at sea shores
    - phases of the moon
    - seasonal changes in weather
    - annual flooding of the Nile

What is Probability? (continued)

- Since pre-historic times, Mankind has also noticed
  - Random phenomena
    - results of coin tosses
    - results of rolling dice
    - results of horse races
- Legends and folk tales from all over the world refer to dice games and gambling

Gambling in ancient times

- The Babylonians, Chinese, Egyptians, Greeks, Persians, Romans, Vikings, the Aryans in ancient India, all gambled
- Aristotle discussed dice probabilities
- The Indian epic ***The Mahabharata*** tells of a dice game between rival clans in which one side uses loaded dice to gain an unfair advantage

Gambling in ancient times

- Curiously, the Old Testament does not include any mention of gambling — the New Testament does
- However, the Israelites were aware of probabilistic ideas
- Choosing by lots, or casting lots, i.e., making a random choice among all the possibilities, has been recognized from ancient times as a fair method

Gambling in modern times

- Modern mankind also loves to gamble
- Government prohibitions against gambling are disappearing
- Nowadays, governments even encourage gambling through
  - State-run lotteries
  - Casinos (possibly on riverboats)
  - Betting Parlors, etc

Probability in ordinary language

- Probabilistic notions are commonplace in everyday language usage
- We use words such as
  - probable/improbable; possible/impossible
  - certain/uncertain; likely/unlikely
- Phrases such as "there is a 50-50 chance" "the odds are 7 to 4 against" "the probability of precipitation is 20%" are understood? by most people

Common meanings of Probability

- What do most people understand about probabilities?
- What is the probability that a coin comes down Heads when it is tossed?
- Almost everyone answers $^1/_2$ Why $^1/_2$?
- "Because there are two sides to the coin"
- "Because when tossed repeatedly, the coin will turn up Heads half the time"

Classical Approach to Probability

- "The probability of Heads is $^1/_2$ because there are two sides to the coin"
- This is called the classical approach to probability
- More generally, if there are n possible outcomes of an experiment, then each outcome has probability $^1/_n$
- Justification: Symmetry principle; principle of indifference (or insufficient reason)

Problems with Classical Approach

- What exactly is an outcome?
- If we toss two coins, are there three outcomes or four outcomes?
  - {0 Heads, 1 Head, 2 Heads}?
  - {(T,T), (T,H), (H,T), (H,H)}?
- Note that 2 Heads has probability $^1/_3$ or $^1/_4$ depending on the choice
- "There are only two outcomes: either I win the Lottery, or I don't, so probability is $^1/_2$?"

Relative Frequency Approach

- "The probability of Heads is $^1/_2$ because when tossed repeatedly, the coin will turn up Heads half the time"
- How do you know that the coin will turn up Heads half the time?
- Suppose multiple tosses have resulted in 50% Heads. Setting P(Head) = $^1/_2$ is the relative frequency approach to probability

Relative Frequency Approach

- If an outcome x occurs m times on N trials, its relative frequency is $^m/_N$ and we define its probability P(x) to be $^m/_N$
- Does there exist a probability of Heads for a mint-new untossed coin?
  Or do probabilities come into existence only after multiple tosses?
- How large should N be?
  Are probabilities re-defined after each toss?

Probability as beliefs

- Many assertions about probability are essentially statements of beliefs
- A fair coin is one for which P(Heads) = $^1/_2$ but how do we know whether a given coin is fair?
- Symmetry of the physical object is a belief
- That further tosses of a coin for which P(Heads) = $^1/_2$ will result in 50% Heads is a belief

Subjective Approach to Probability

- Statements about probabilities are statements about our beliefs
- Different persons can observe the same phenomenon and arrive at different beliefs
- Many bets on sporting events arise from differences in beliefs!
- Probabilities are often expressed in terms of odds in such cases

Subjective Approach to Probability

- Odds of m to n against an event means the probability of the event is n/(n+m)
- Odds describe payoffs from bets: if you bet $n that the event will occur, you will either gain $m or lose $n
- Over a long series of trials, your average gain will be zero
- Different assessments of the odds lead to the belief that one has an advantage

Probabilities as numbers

- We always use numbers in the range [0,1] to denote probabilities
- Are probabilities merely numbers without any additional meaning ascribed to them?
- This is called the axiomatic approach to the probability calculus
- The axiomatic approach is consistent with all the approaches described above

Axiomatic Approach to Probability

- In the axiomatic approach, probabilities are numbers in the range [0,1]
- Certain probabilities are assumed to be given (we don't ask how!)
- The axiomatic approach allows the calculation of other probabilities in a mathematically and logically consistent manner
- It is a probability calculus

Axiomatic Approach to Probability

- The axiomatic approach allows the computation of probabilities without requiring philosophical discussions about the meaning of probability
- If you believe that probability is purely subjective, then the axiomatic approach tells you what other beliefs are logically consistent with your chosen beliefs

Axiomatic Approach to Probability

- The axiomatic approach to probability will be used throughout this course
- We will build a mathematical model of probabilities that will allow us to draw useful and interesting conclusions about the real world
- The model does not describe all aspects of probability but enough aspects are captured to make the model very useful

A homily about models

V = IR

- How well does the model shown above describe a real resistor?
- Mathematical (or physical) models of real-world phenomena do not necessarily capture all the features of reality

What is/are Statistics?

- Statistics are collections of data
- Statistics is the science of
  - collecting data
  - organizing data
  - analyzing data
  - making inferences from data
- In this course, we shall study a few selected topics in statistical inference

Probability versus Statistics

- Probability theory deals with questions such as
  "What is the probability that Heads shows six times on ten tosses of a fair coin?"
- Whether the coin actually is fair is not open to question — fairness is one of the givens in the problem
- The probability question can be answered precisely and unambiguously

Probability versus Statistics

- Statistical inference deals with questions such as
  "Given that Heads showed on six of ten tosses, is it reasonable to assume the coin is fair?"
- A definitive answer is not possible since different people have different ideas as to what is reasonable

Probability versus Statistics

- Statistical inferences are accompanied by levels of confidence
- "Yes, I am 95% sure the coin is fair"
- "Yes, I am 99% sure the coin is fair"
- NO statistical procedure can prove or disprove that the coin is fair
- Fairness is a matter of belief, and the degree of belief is the confidence level

Summary

- Some elementary ideas of probability and statistics have been introduced at a fairly intuitive level
- Various approaches to probability have been described briefly
- The notion of statistical inference has been introduced
- But what does all this have to do with electrical and computer engineering?