University of Illinois at Urbana-Champaign

ECE 313: Probability with Engineering Applications

Hour Exam I: Fall 2000

October 2, 2000

One Hour

Problem 1 *(20 points)* Let X be a geometric random variable with unknown parameter p. If it is observed that X took on the value ten, what is the maximum-likelihood estimate, $\hat{p}$ for the parameter p?

Problem 2 *(25 points)* A small island has decided to select a new monarch. The set of candidates for this position is { Andy, Beth, Chuck, Di, Eddie, Fergie }. The selection process begins with a search committee selecting a short-list, S, of three candidates from this set. Assume that all short-lists are equally likely to occur.

Part (a) 5pts. What is the probability that Chuck will be on the short-list?

Part (b) 10pts. Once a short-list has been selected, a general election will be held to select the monarch from this short-list. Suppose that the general population does not like Chuck, and therefore they will not elect him, even if his name appears on the short-list. Assume that if Chuck is on the short-list the other two non-Chuck candidates are equally likely to be elected, and that if Chuck is not on the short-list then all three candidates are equally likely to be elected.
What is the probability that the new monarch will be female?
(Note: the set of female candidates is { Beth, Di, Fergie }.)

Part (c) 10pts. Under the same conditions as for part (b), what is the probability that Beth will be the new monarch?

Problem 3 *(10 points)* Consider four independent trials of the experiment of rolling a fair die. Let X_1 be the sum of the numbers showing on the first two rolls, and let X_2 be the sum of the numbers showing on the last two rolls. What is the conditional probability that the numbers X_1 and X_2 are adjacent on a clock face, given that $X_1 > 10$? You may leave the denominator as a power of six.

Problem 4 *(15 points)*

Let A and B be *disjoint events* with $P(A) = 1/2$ and $P(B) = 1/6$. If $P(C|A) = 1/7$ and $P(C|A \cup B) = 3/14$, what is $P(C|B)$?

If the answer cannot be determined, check the box on the left, and leave the right hand box blank.

Problem 5 *(30 points)* In a certain game of chance, you roll one fair die. If you roll a six, you win ten dollars; otherwise, you lose two dollars. Let X be a random variable equal to the number of times you roll a six, and let the random variable Z denote the amount of your winnings. For example, if you play ten times and you roll a six five of those times, your winnings will be \$40. Suppose that the outcome for each roll of the die is independent of all other rolls.

Part (a) 9pts. Assume that you play the game ten times. Determine $E[X^2]$.

Part (b) 8pts. If you play the game ten times, what is $E[Z]$?

Part (c) 5pts. Suppose you play the game until you win once (i.e., you play repeatedly until you roll a six). Let the random variable W be the number of the round on which you finally win. Determine $E[W]$.

Part (d) 8pts. Suppose you play the game until you have rolled a six five different times. What is the expected value of your winnings?