

The morning half of the Ph.D. Qualifying Examination taken by ECE graduate students consists of problems from courses required for the Bachelor's degrees in EE and CompE. Each problem is worth 10 points. The following problem appeared on the exam (held last Saturday) this semester.

- 9.** The number of hours \mathbf{R} that a student spends reading about probability theory in preparation for the Qualifying Examination and the number of hours \mathbf{S} that the student spends sleeping can be modeled as random variables with joint probability density function

$$f_{\mathbf{R},\mathbf{S}}(x,y) = \begin{cases} K, & 10 \leq x + y \leq 20, x \geq 0, y \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) **(1 point)** What is the value of K ?

Unfortunately, the more the student tries to read about probability theory, the more confused the student gets. Also, the less the student sleeps, the more tired the student gets. As a result, the student's score \mathbf{T} on Problem 9 is related to \mathbf{S} and \mathbf{R} via the equation

$$\mathbf{T} = 5 + 0.25(\mathbf{S} - \mathbf{R}).$$

- (b) **(3 points)** What is $E[\mathbf{T}]$, the expected value of \mathbf{T} ?
- (c) **(3 points)** What is $P\{\mathbf{T} \geq 7.5\}$?
- (d) **(3 points)** The student's score on Problem 9 is 7.5. What is $P\{\mathbf{R} \leq 1 \mid \mathbf{T} = 7.5\}$, the conditional probability that the student spent at most one hour reading about probability theory in preparation for the Qualifying Examination?

Not all parts of this problem have been covered in class as yet, but you should be able to do parts (a) and (c), as perhaps part (b) as well. You might want to try the following as well.

- (e) What are the marginal pdfs of \mathbf{R} and \mathbf{S} ?
- (f) Find $P\{\mathbf{R} + \mathbf{S} \leq 15\}$.
- (g) More generally, find $P\{\mathbf{R} + \mathbf{S} \leq u\}$ for all real numbers u and hence determine the pdf of the random variable $\mathbf{U} = \mathbf{R} + \mathbf{S}$.
- (h) Find the pdf of the random variable \mathbf{T} defined above. Hint: it is much easier to figure out the CDF of \mathbf{T} and then find the pdf.

Solutions to the above problem will be posted on Friday.